For Degree Students

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- So if we see something as **WHITE**, that means … It reflected back all the wavelengths of light to our eyes
-
- If we see something as **RED** or **BLUE**

  It reflected only the **RED** or only the **BLUE** wavelengths

  The others were absorbed.
- And if we see something as **BLACK** ?

  It did not reflect back any of the light.

That means the BLACK substance will always active for only catching the light ?

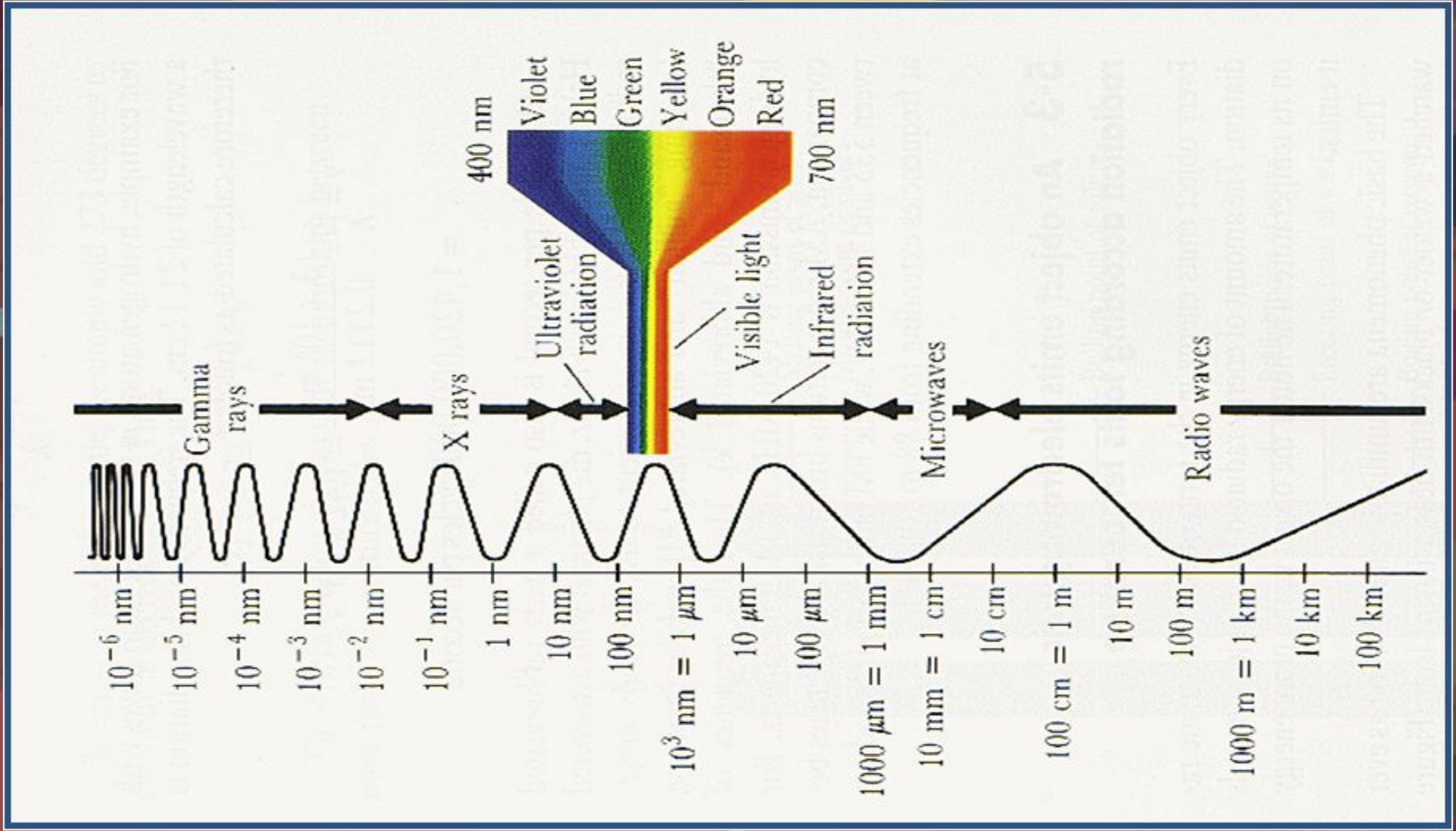
What actually happens?

§ The black body

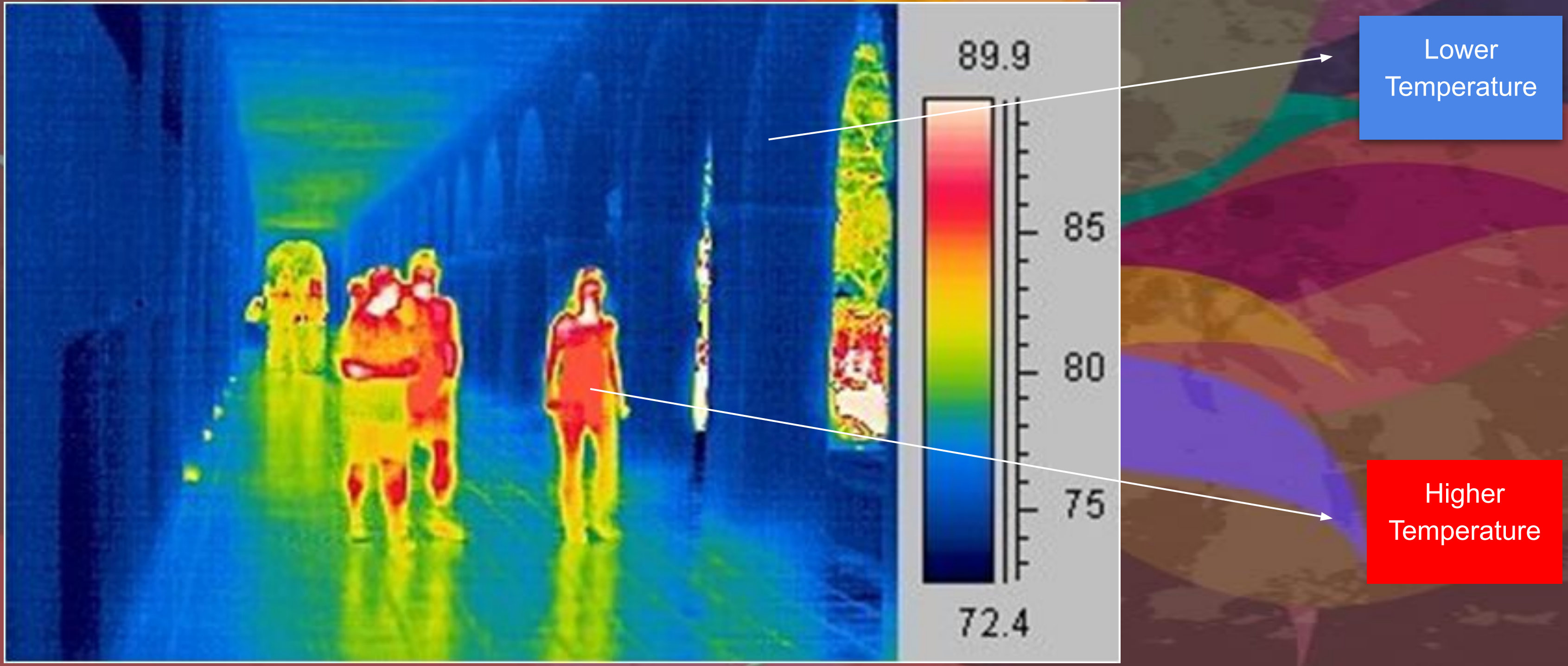
A perfectly black body is one **which absorbs totally all the radiations of any wavelength which fall on it**.
Since it neither reflects nor transmits any radiation it appears black whatever the colour of incident radiation may be.

On the other hand **when such a body is heated it emits radiations of all possible wavelengths**.

A part of all possible wavelengths



Every body can radiate like a black body up to some extent.



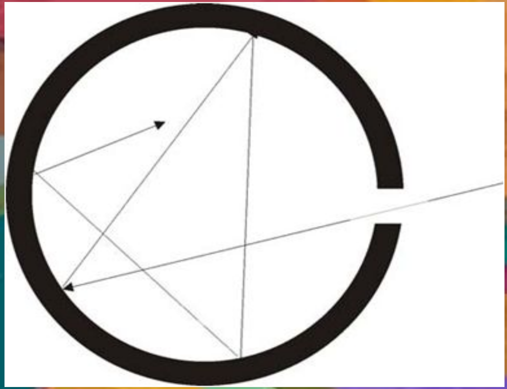
Radiation from a black body

When a black body is placed inside a uniform temperature (isothermal) enclosure, it will emit the full radiation of the enclosure after it is in equilibrium with the enclosure.

These radiations are independent of the nature of the substance, nature of the walls of the enclosure, and presence of any other body in the enclosure

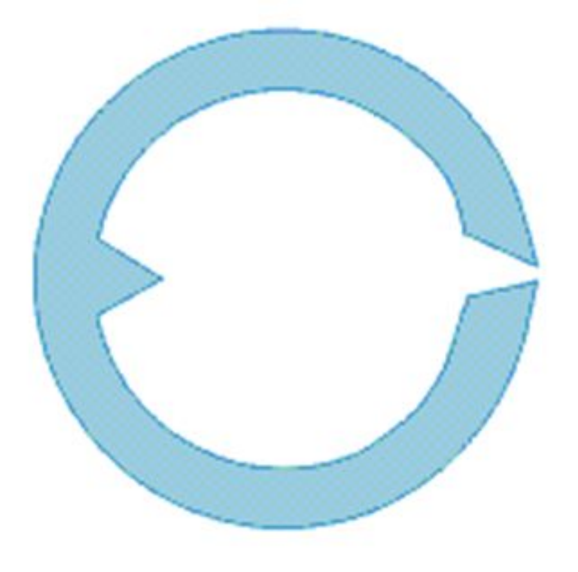
It depends only on temperature. Such radiations in a uniform temperature enclosure are known as black body radiations.

§ The implementation of a black body in practice

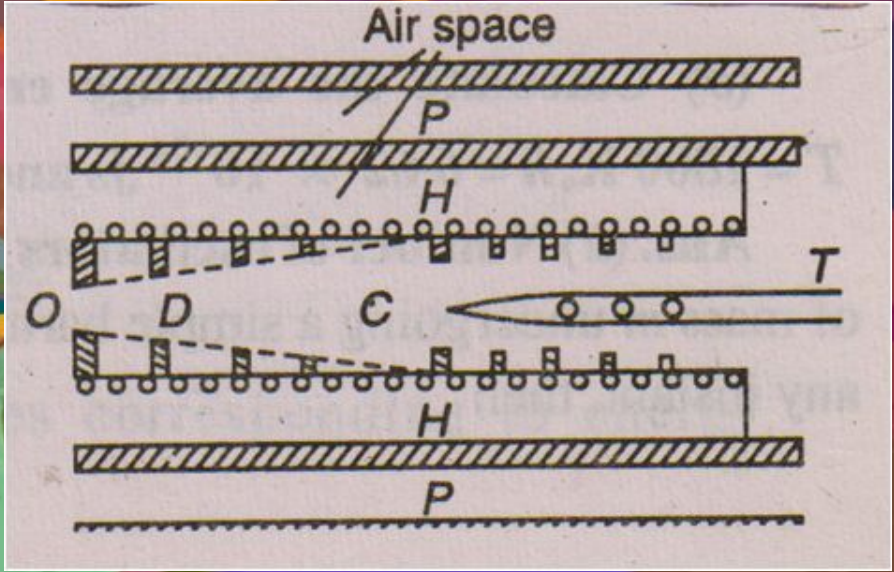


The Ulbricht sphere, is a sphere with a small opening, where only a small amount of radiation can escape, so that the interior of the sphere is in thermal equilibrium with the walls, which are kept at a constant temperature. The inside walls are typically made of diffuse material, so that after multiple scattering of the walls any incoming ray is absorbed, i.e. the wall opening is black

Other well known Implementations



Ferry's black body



Wien's black body

Natural black bodies

In the nature all the bodies can absorb and radiate huge kind of wavelength but may not all the radiations like perfect black body.

The radiations coming out of these bodies can be compared with the perfect black body. In some cases these are called **grey bodies**.

§ The emmissivity or emissive power of thermal radiation

Def: Energy radiated into vacuum per sec. per unit area per unit wavelength range from a surface is called emissive power (e_λ) [$\Rightarrow$ Energy radiated into vacuum per sec. per unit area within wavelength λ and $\lambda + d\lambda$ is $e_\lambda d\lambda$]. For a black body emissive power is the maximum.

§ The absorptive power

Def: The ratio of energy absorbed to the amount of energy incident is called the absorptive power (a_λ) of the surface.

If dQ is the energy incident per sec per unit area having wavelength within λ and $\lambda + d\lambda$, the amount of energy absorbed per sec per unit area within λ and $\lambda + d\lambda$ is $a_\lambda dQ$.

$\Rightarrow$ Energy reflected or transmitted per sec per unit area within same wavelength range = $dQ - a_\lambda dQ = (1 - a_\lambda) dQ$

§ The Kirchhoff's law of thermal radiation

Energy radiated into vacuum per sec. per unit area within wavelength λ and $\lambda + d\lambda = e_\lambda d\lambda$

and the energy reflected or transmitted per sec per unit area within same wavelength range = $dQ - a_\lambda dQ = (1 - a_\lambda)dQ$

$\therefore$ The total energy given out per sec per unit area within same wavelength range = $(1 - a_\lambda)dQ + e_\lambda d\lambda$

In the state of thermal equilibrium,

$$\text{Energy incident} = \text{energy given out}$$

$$\Rightarrow dQ = (1 - a_\lambda)dQ + e_\lambda d\lambda$$

$$\Rightarrow a_\lambda dQ = e_\lambda d\lambda$$

Now for a perfect blackbody, $e_\lambda = E_\lambda$ (i.e. emissive power is maximum) and $a_\lambda = 1$ (maximum absorption no reflection)

$$\Rightarrow dQ = E_\lambda d\lambda$$

Eliminating dQ from the general expression by the same for black body,

$$a_\lambda dQ = e_\lambda d\lambda$$

$$\Rightarrow a_\lambda E_\lambda d\lambda = e_\lambda d\lambda$$

$$\Rightarrow a_\lambda E_\lambda = e_\lambda \quad [\because d\lambda \neq 0]$$

$$\Rightarrow E_\lambda = e_\lambda / a_\lambda$$

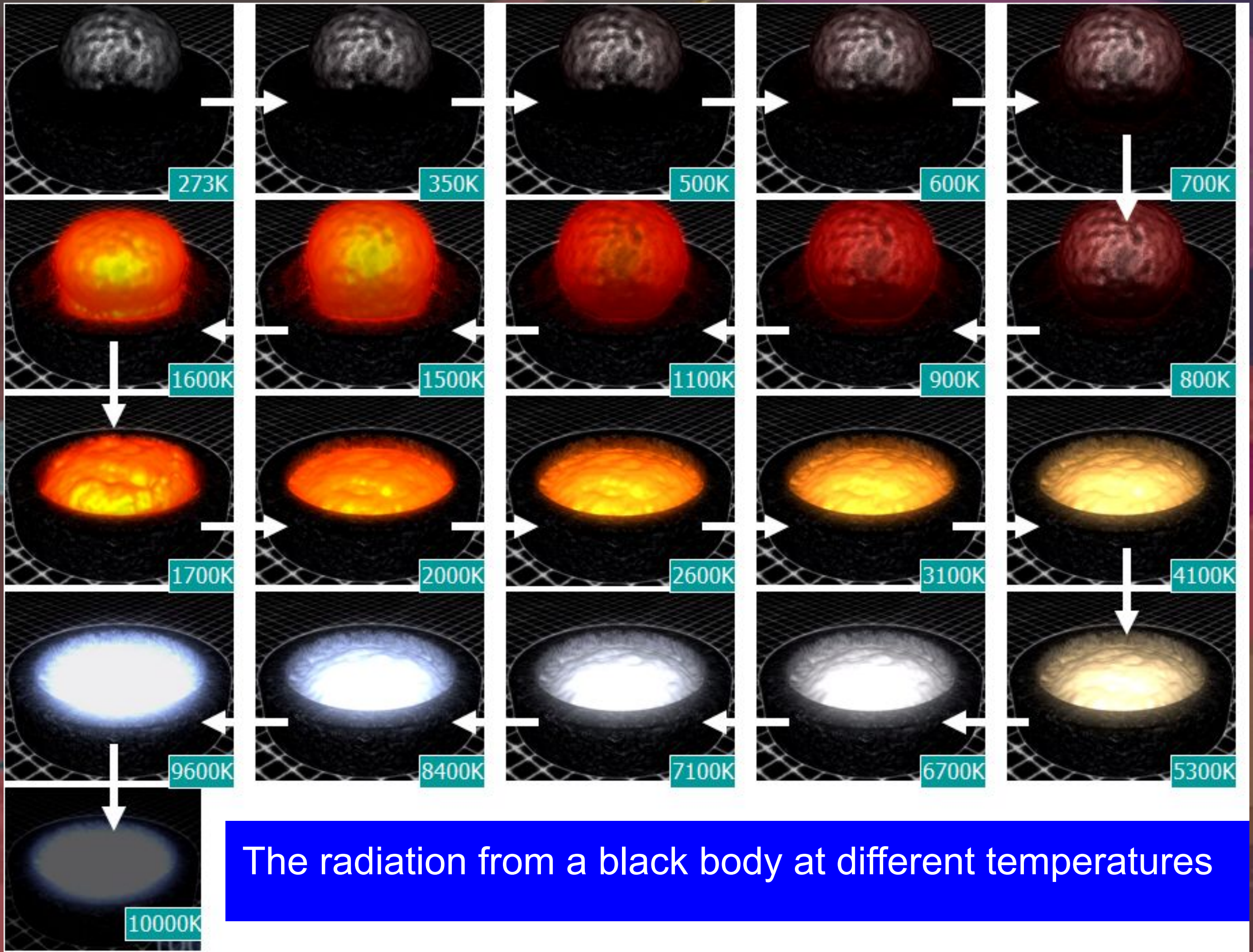
The Kirchhoff's law: The outcome

$$\left(\frac{emissive\ \ power}{absoprtive\ \ power}\right)_{any\ body} = emissive\ \ power\ of\ a\ \ perfect\ black\ body$$

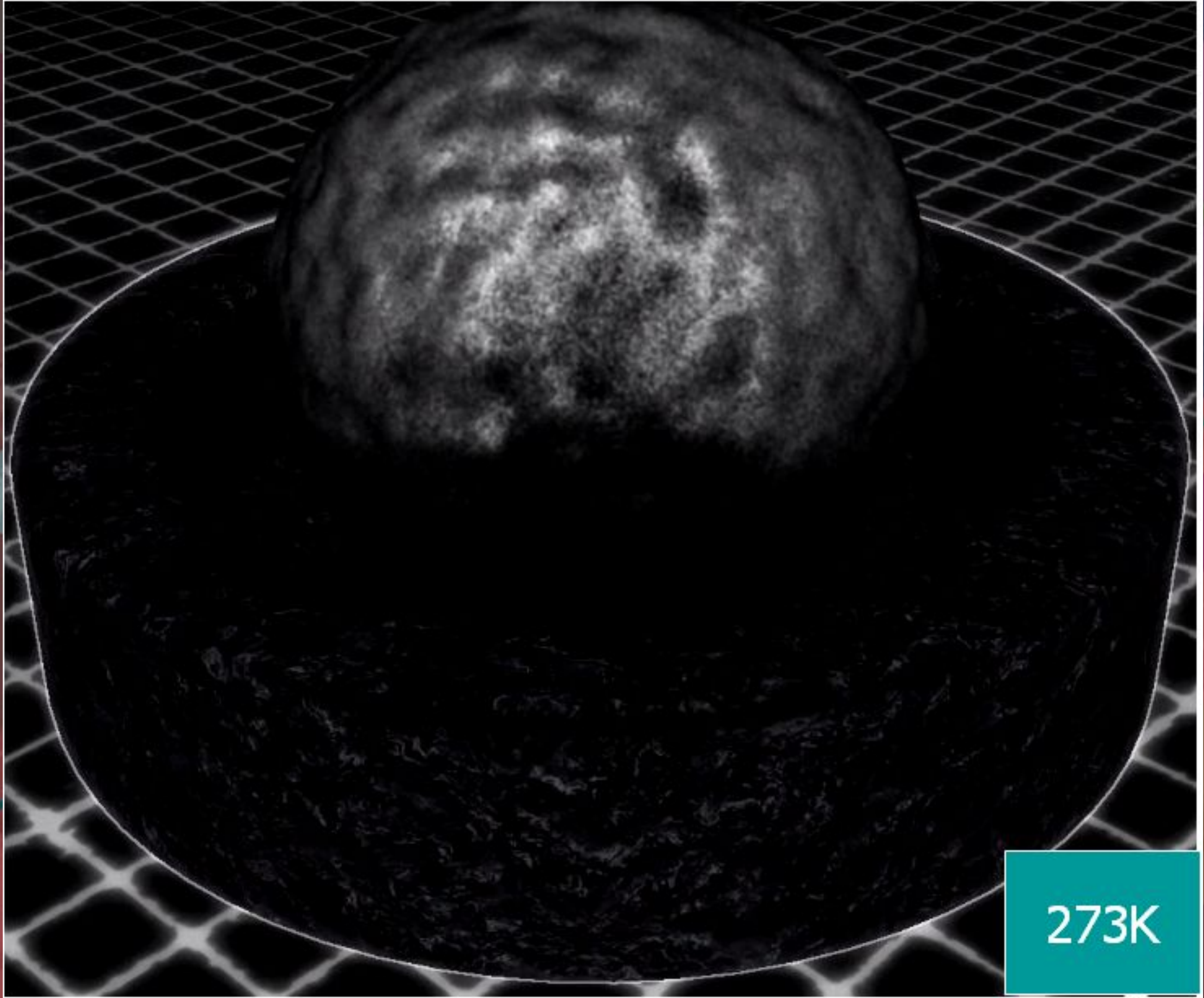
That means there is a way of comparing them and the black body is used as a standard with which the absorption of real bodies is compared.

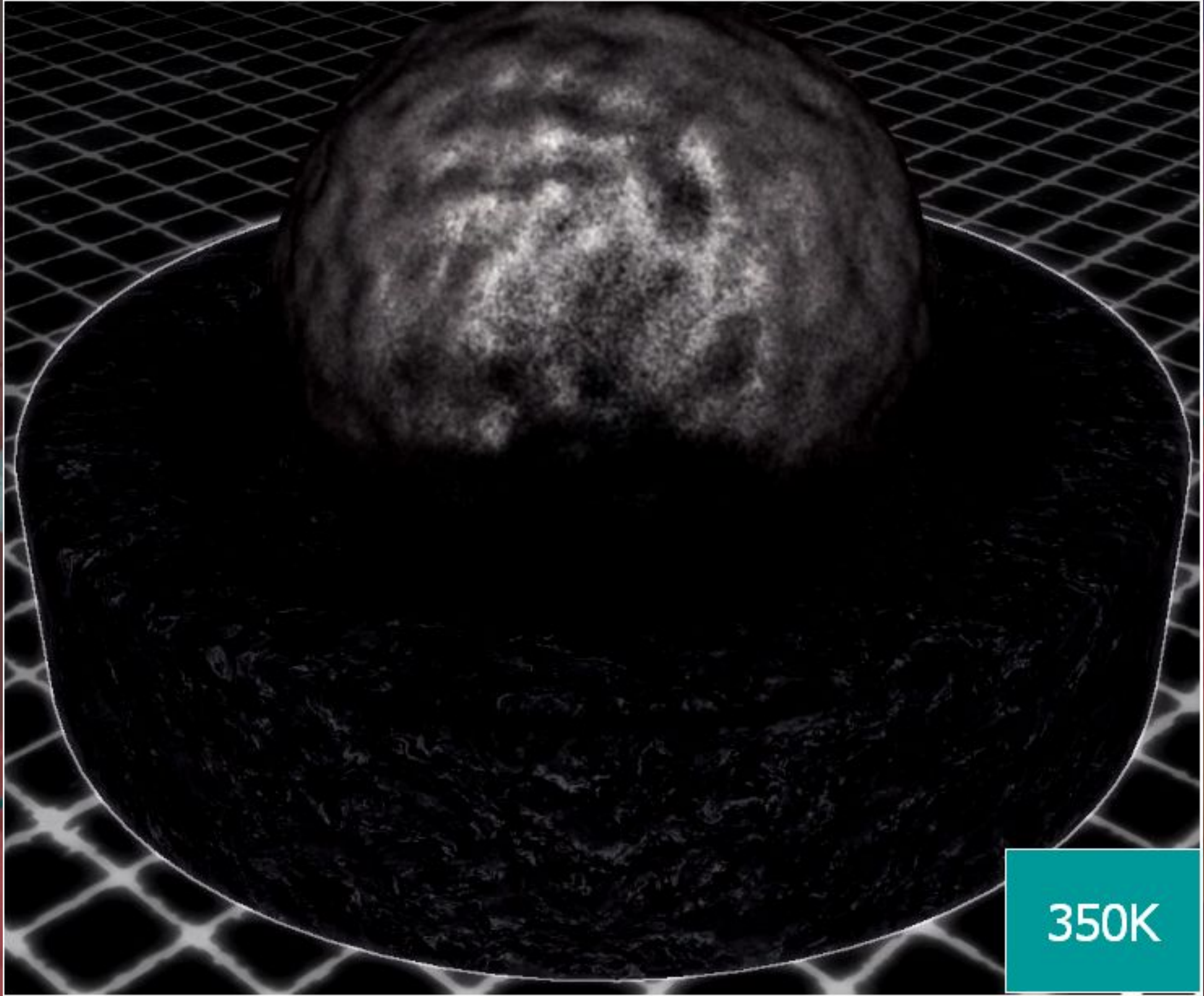
Electrical heating of a blackened Pt-wire

Temperature	Colour
500°C	Dull Red
900°C	Cherry Red
1100°C	Orange Red
1250°C	Yellow
1600°C	White



The radiation from a black body at different temperatures


273K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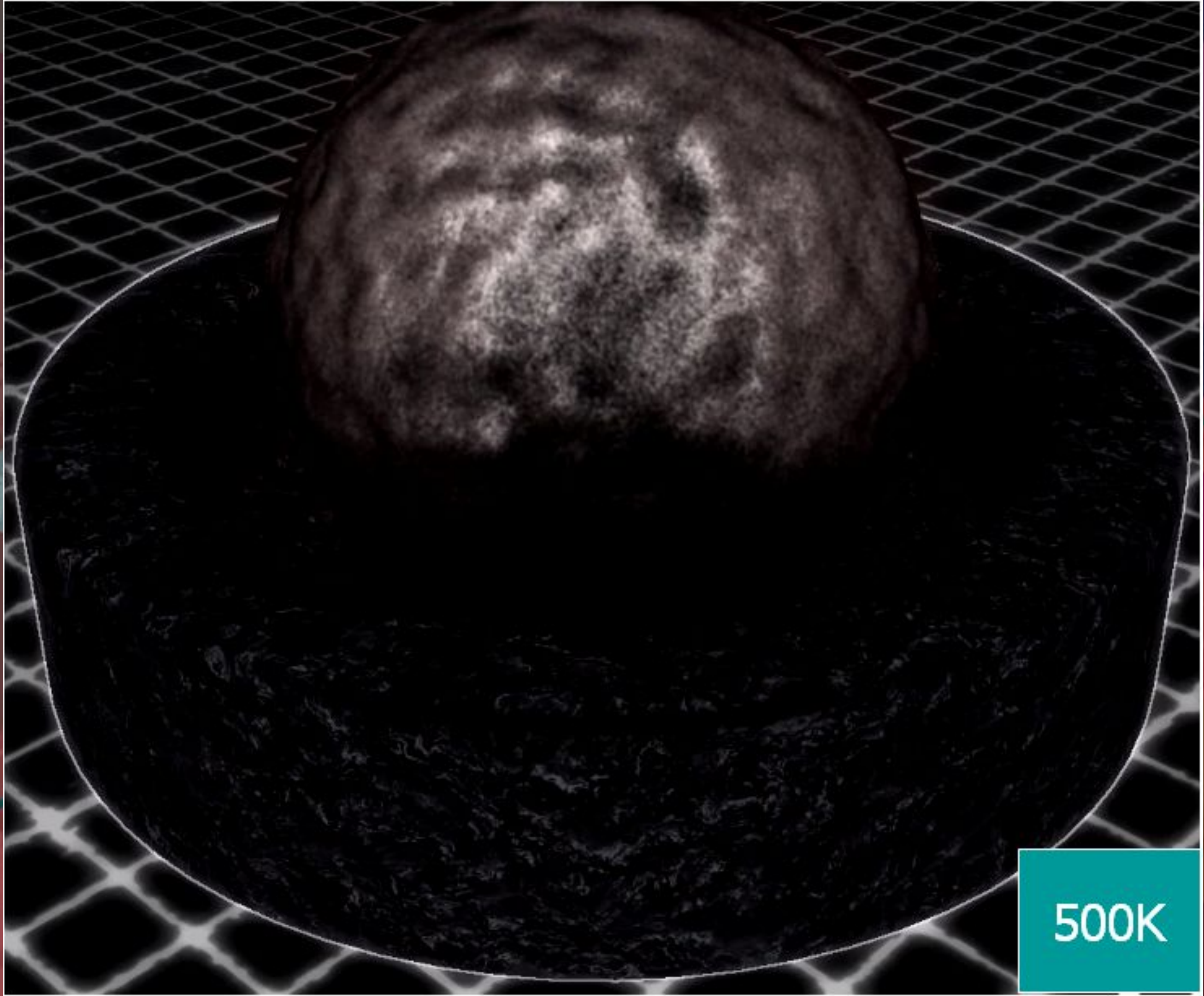
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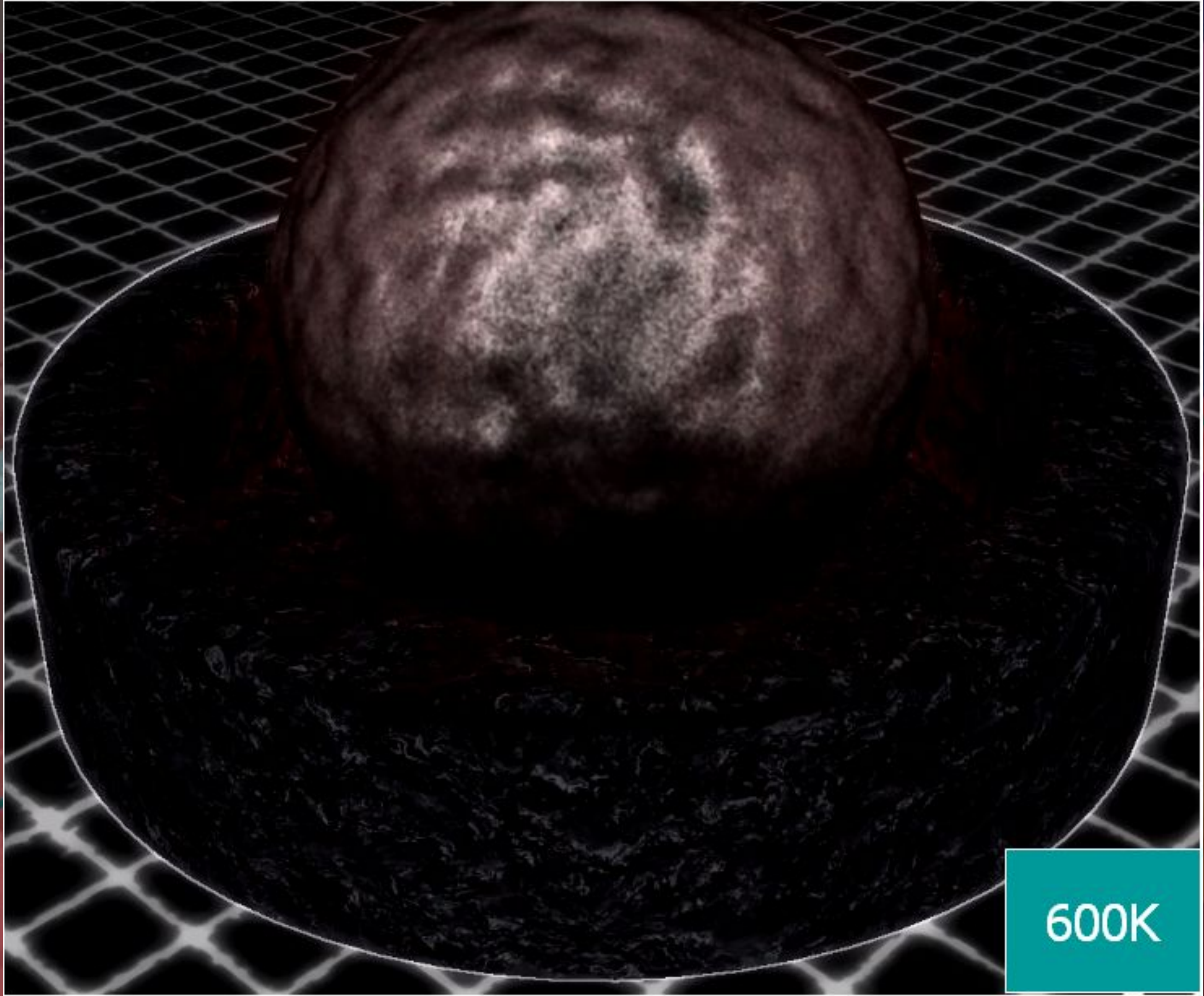
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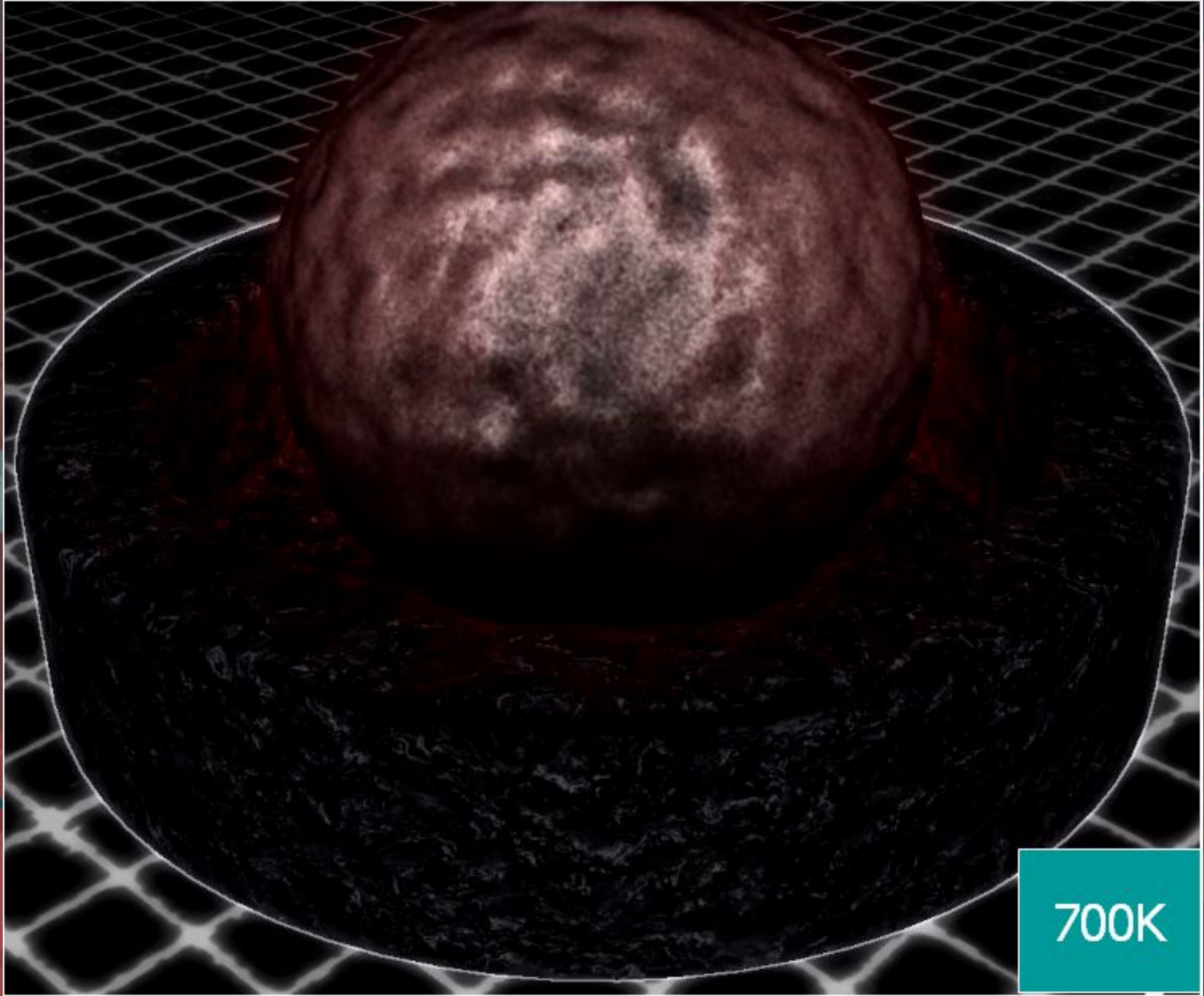
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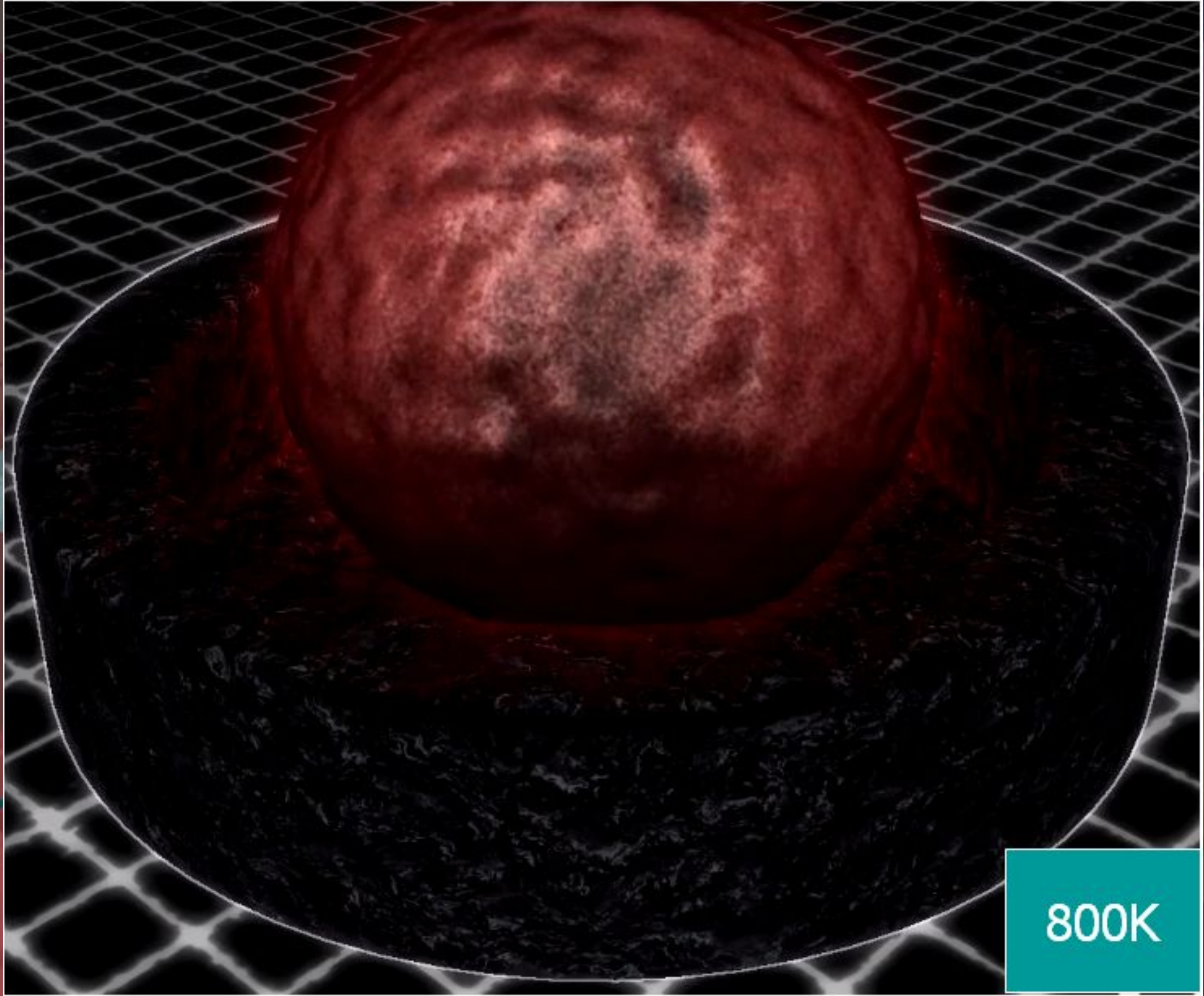
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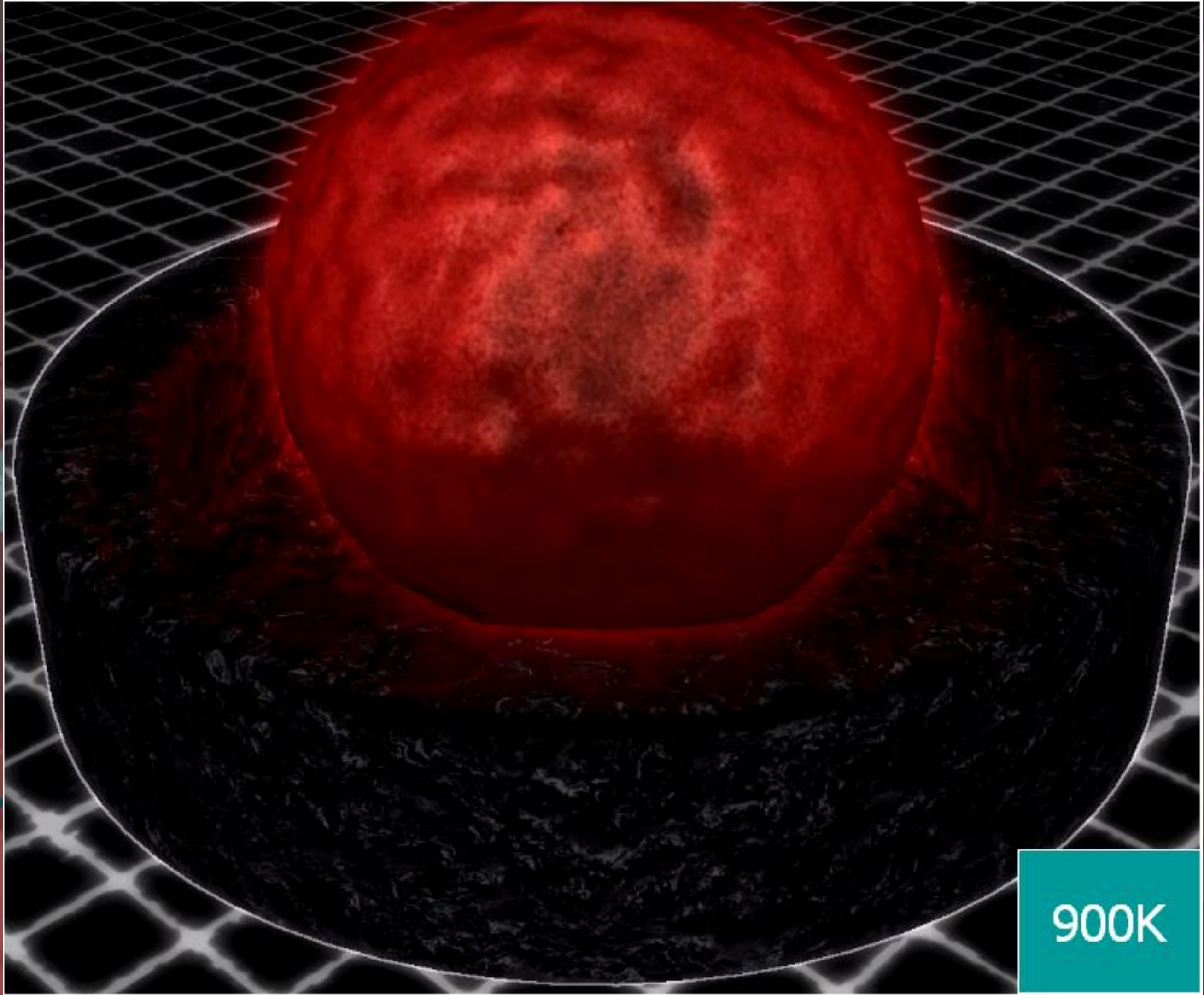
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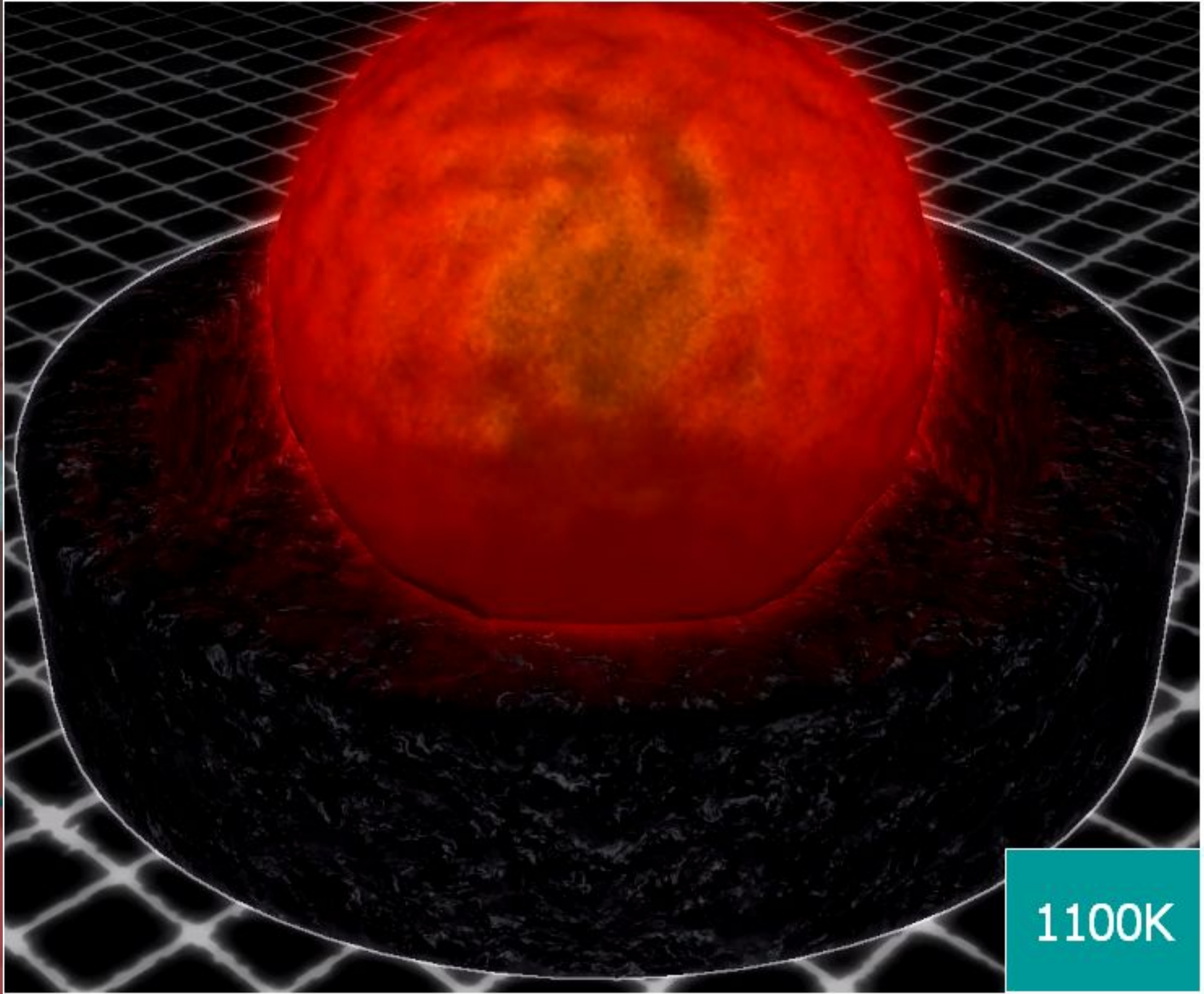
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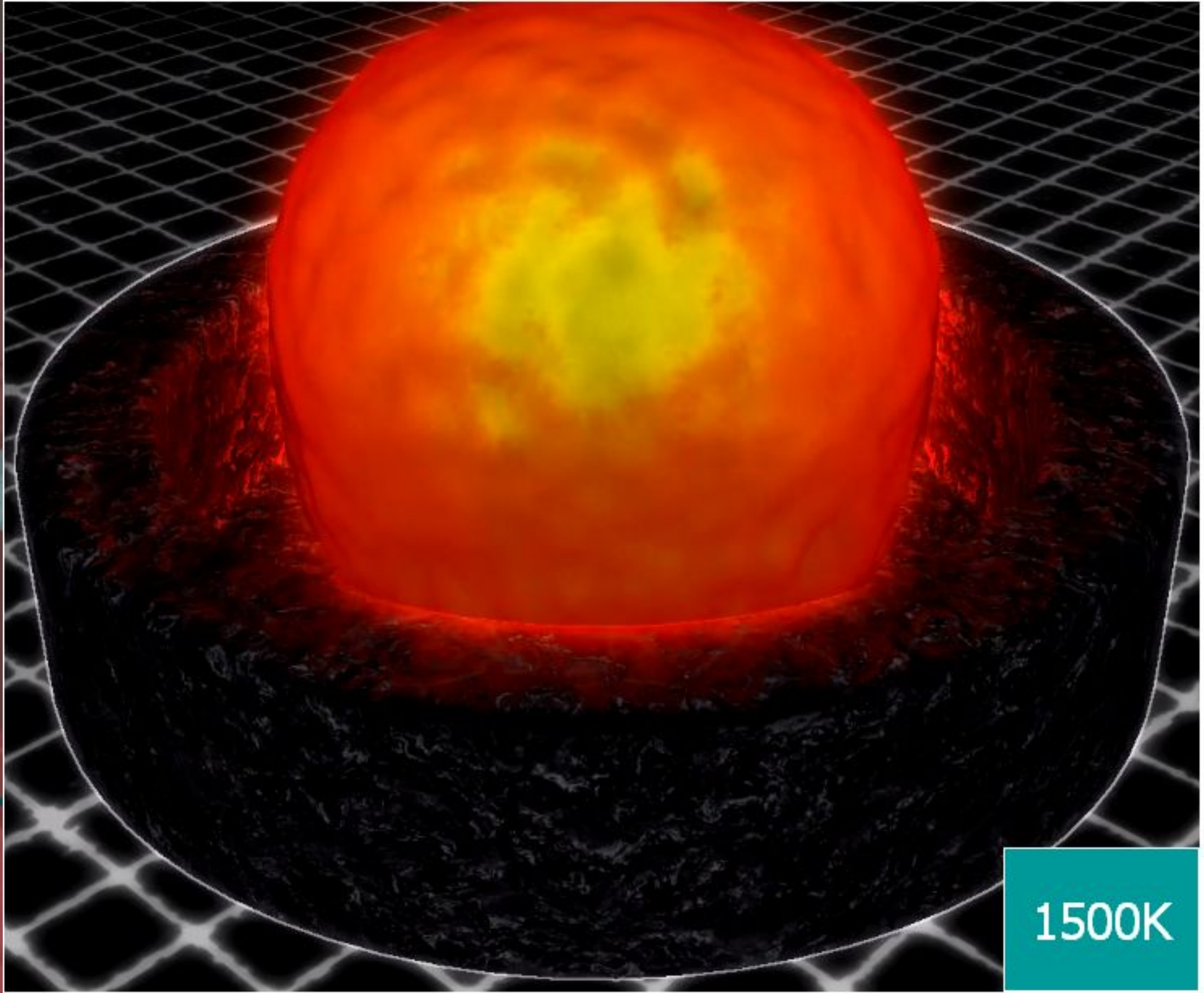
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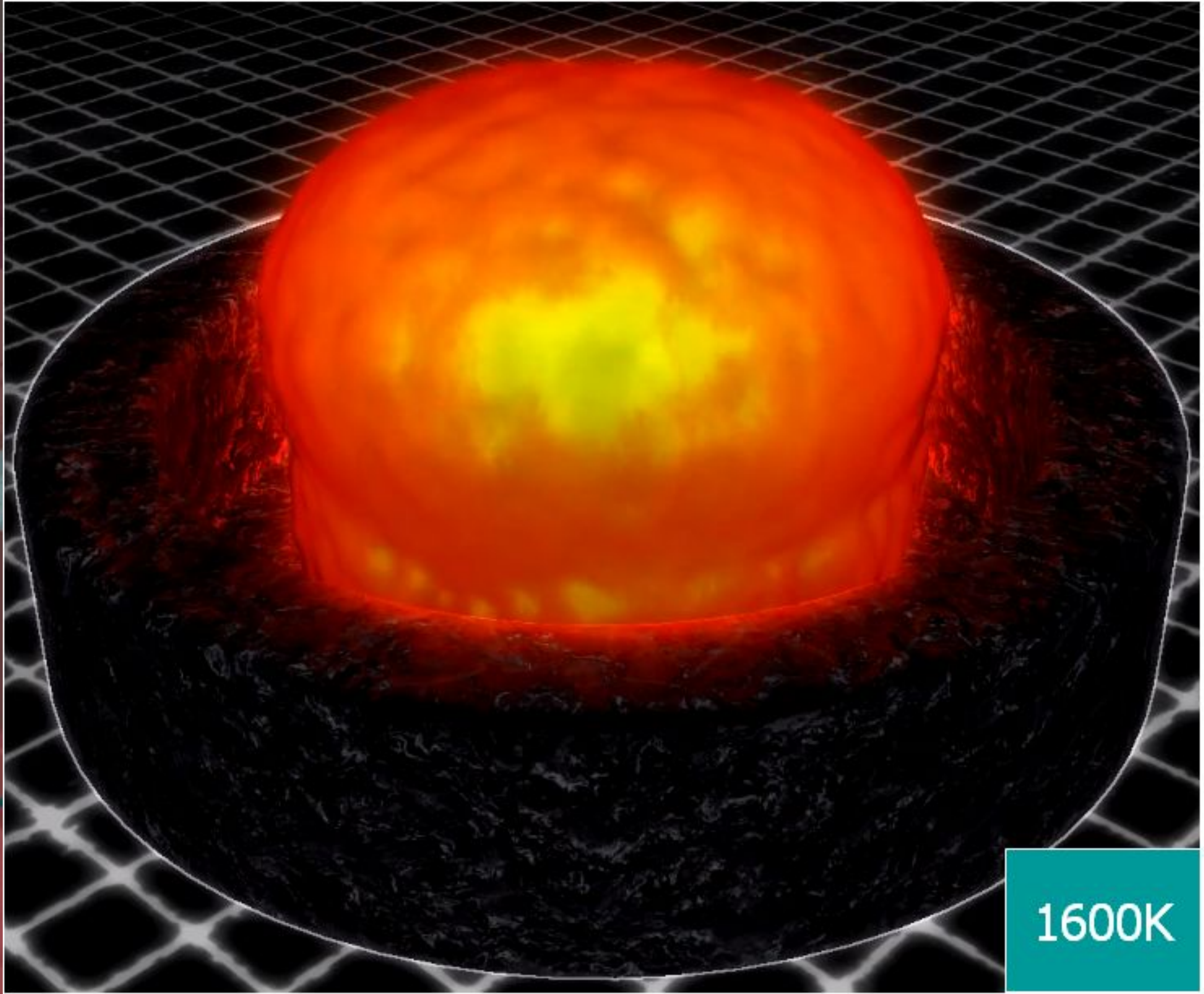
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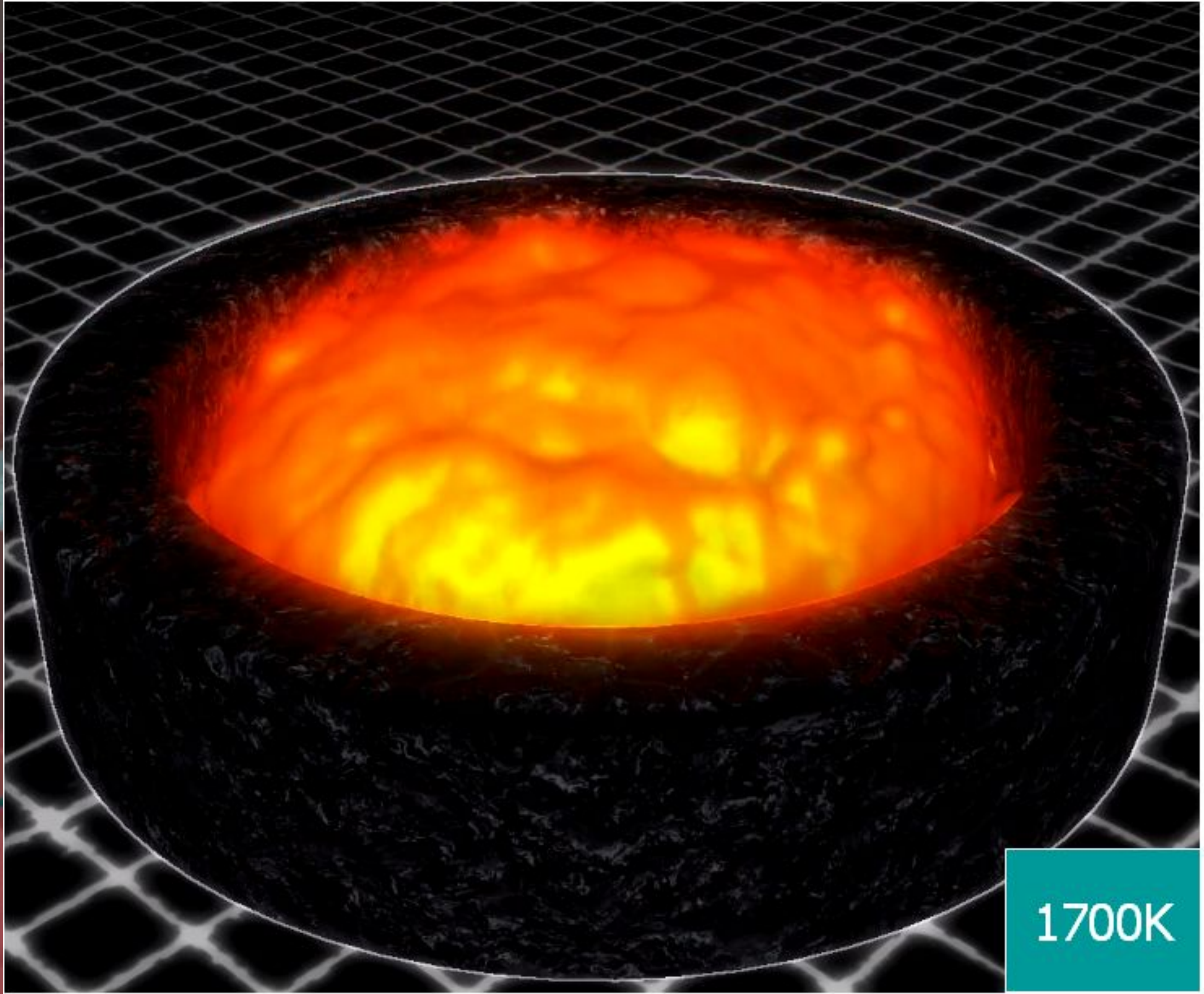
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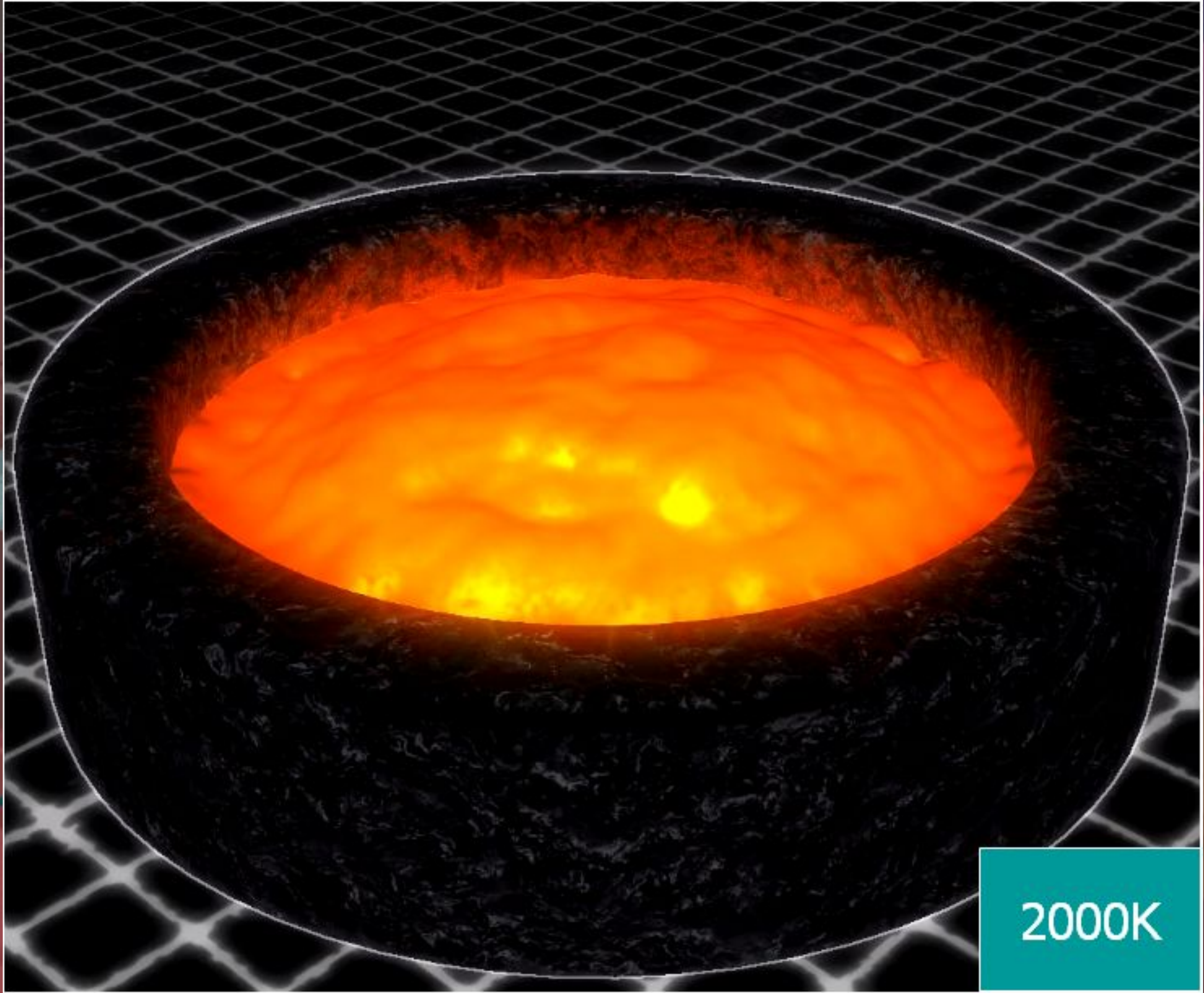
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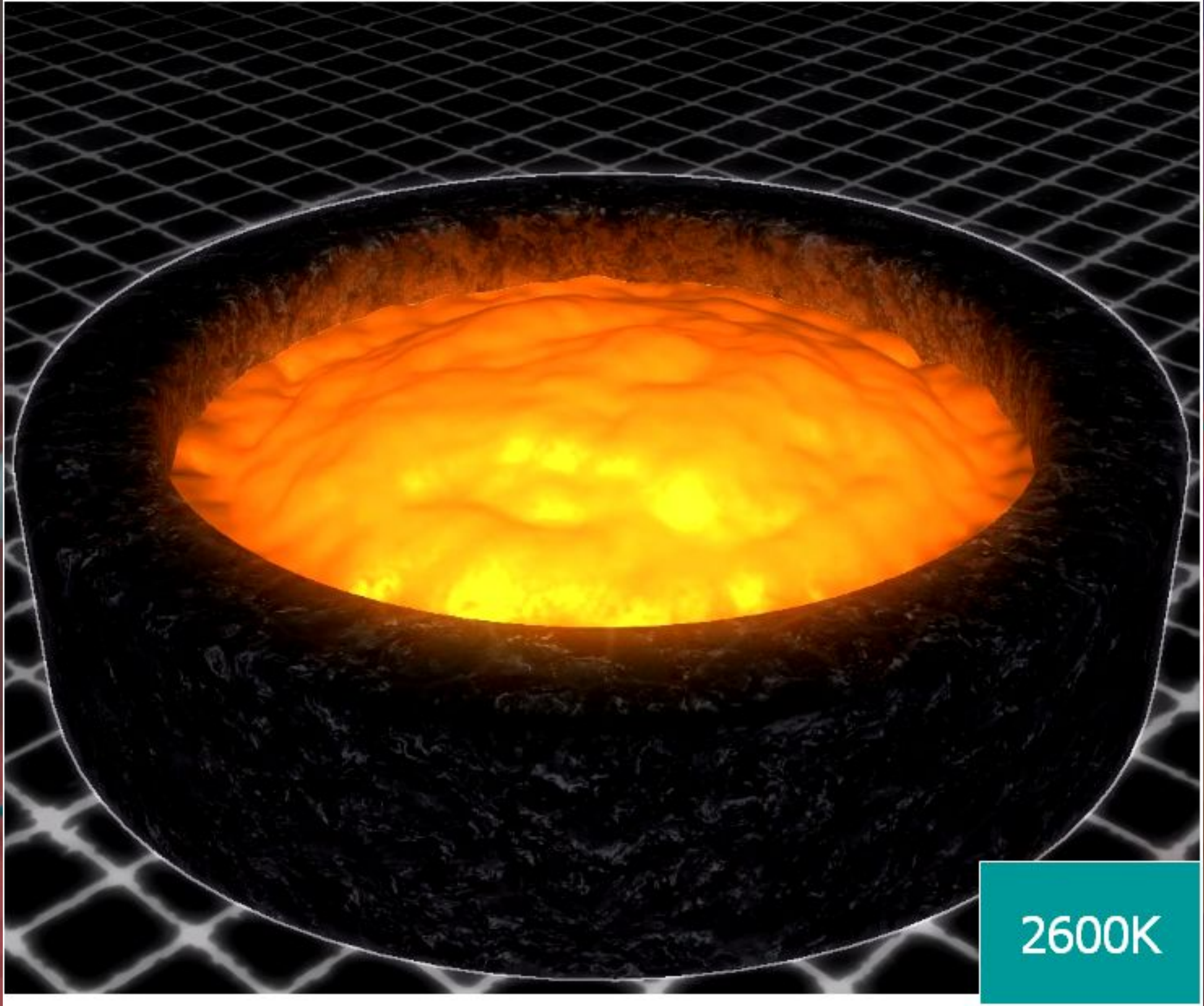
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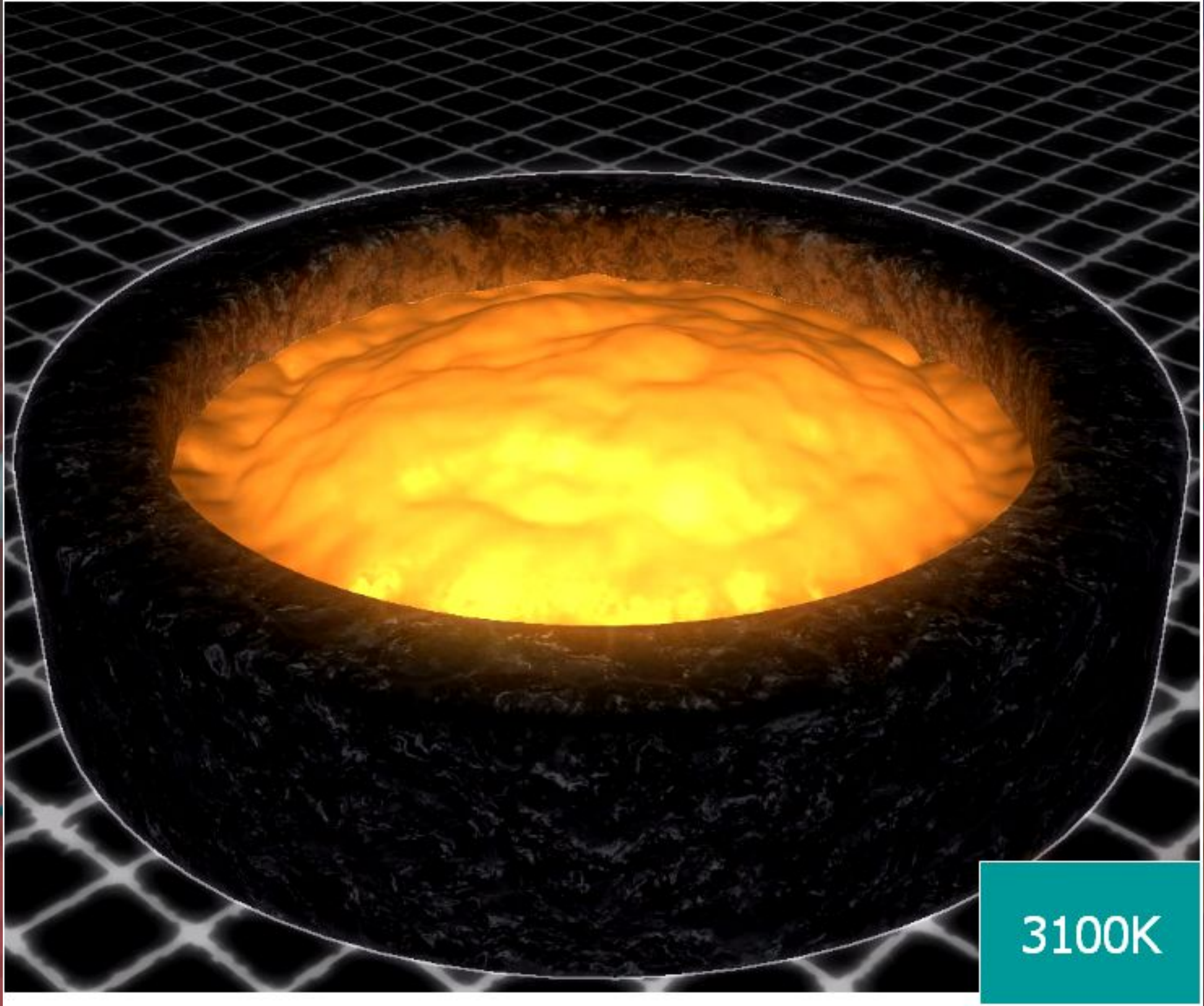
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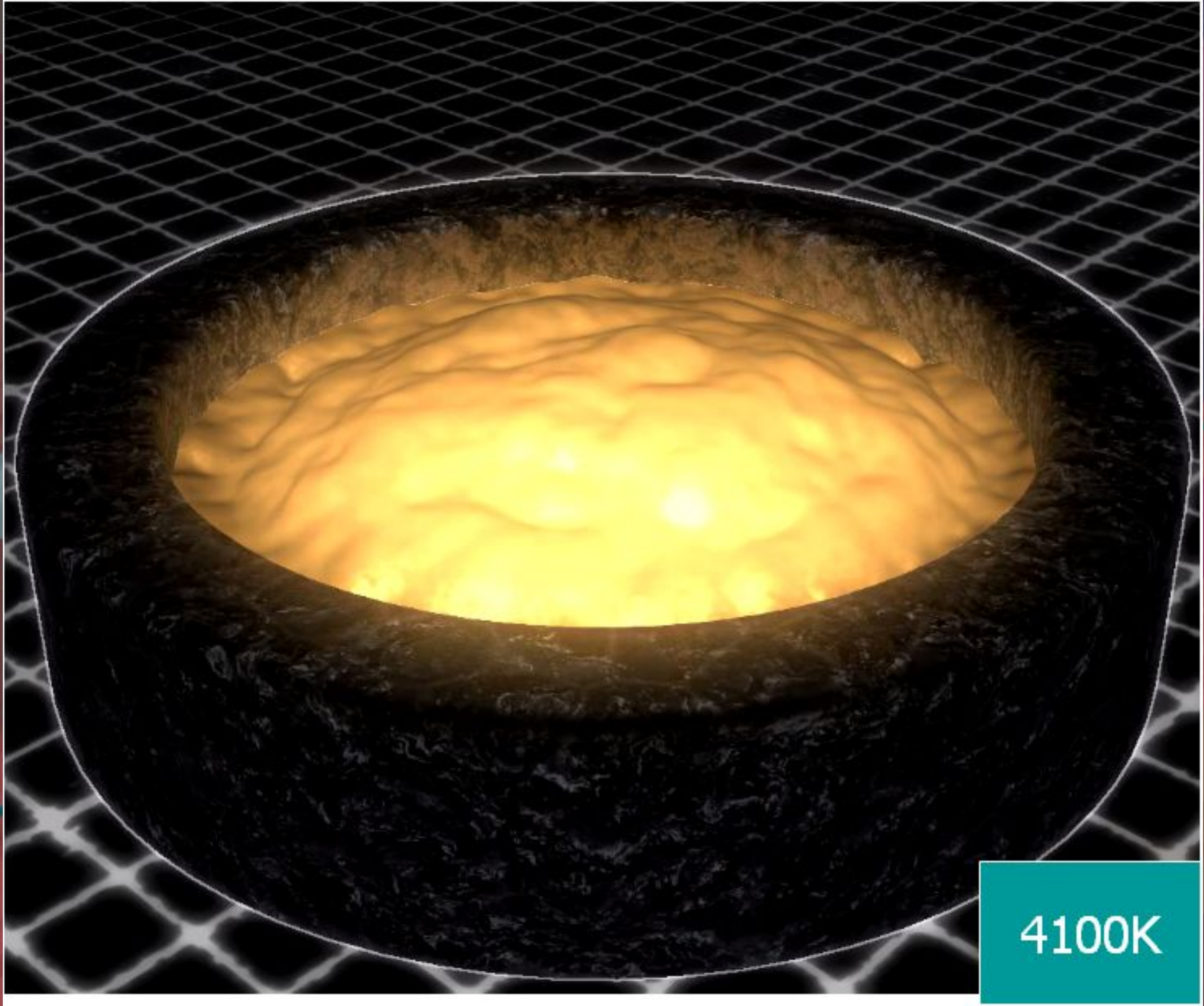
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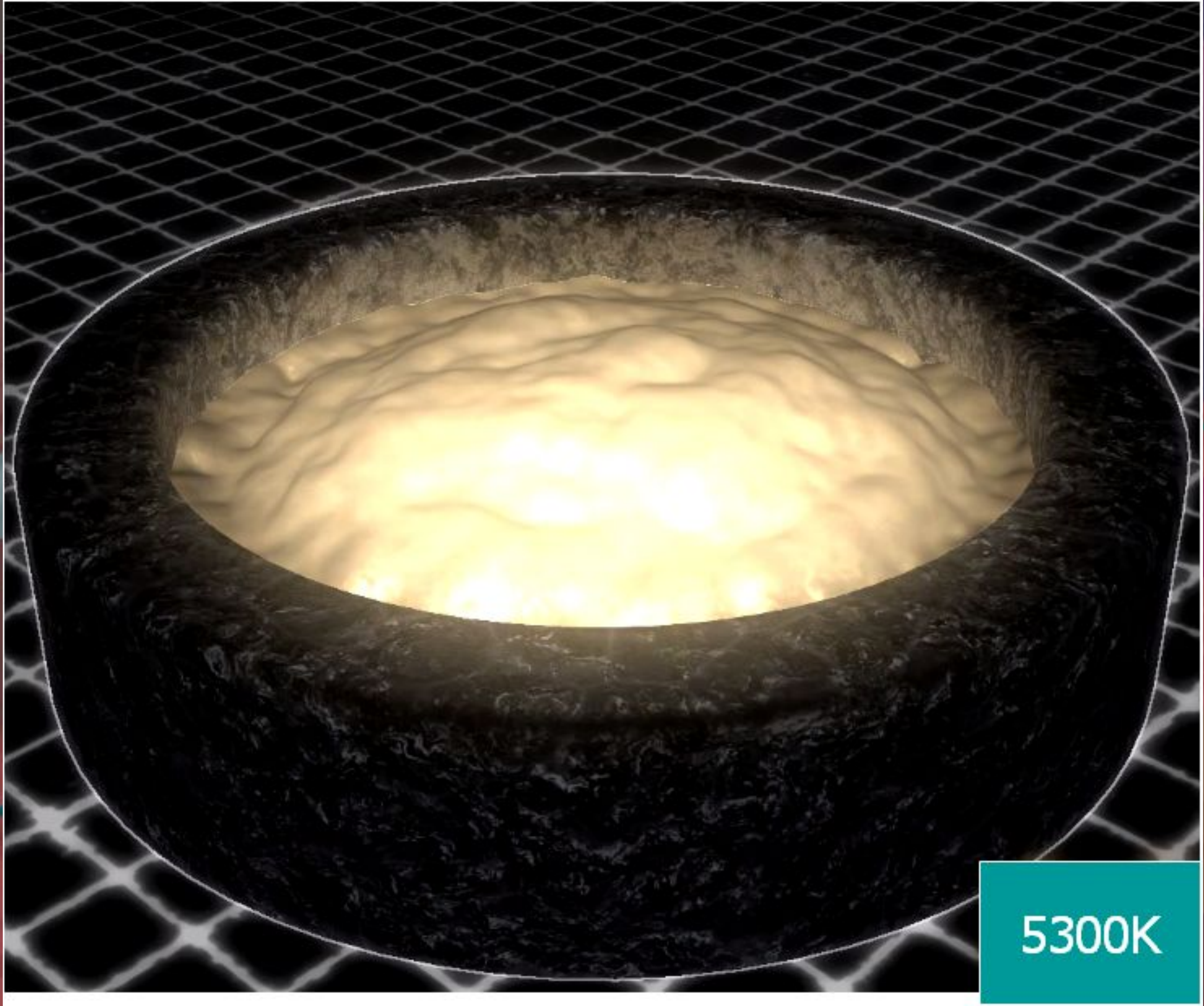
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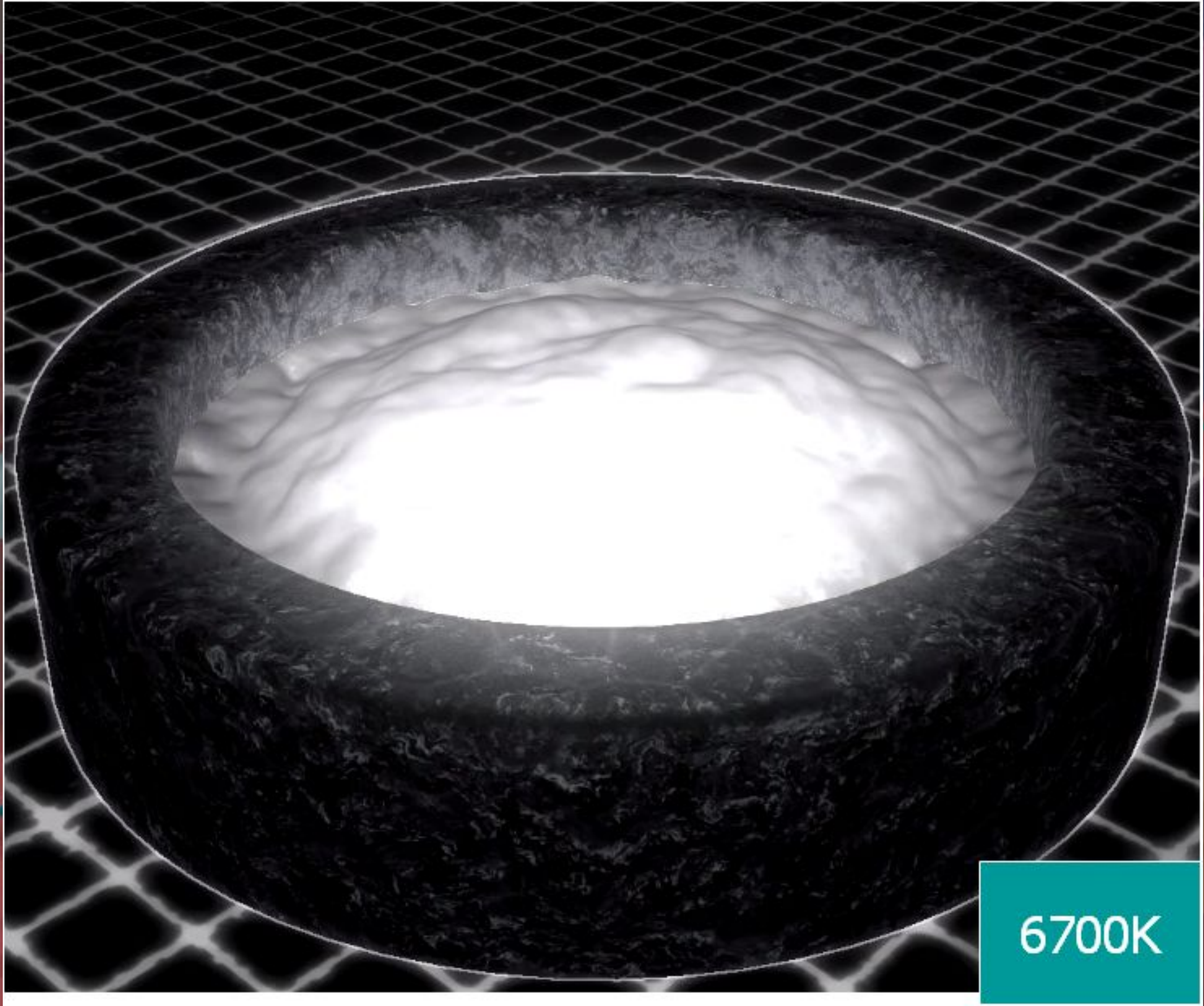
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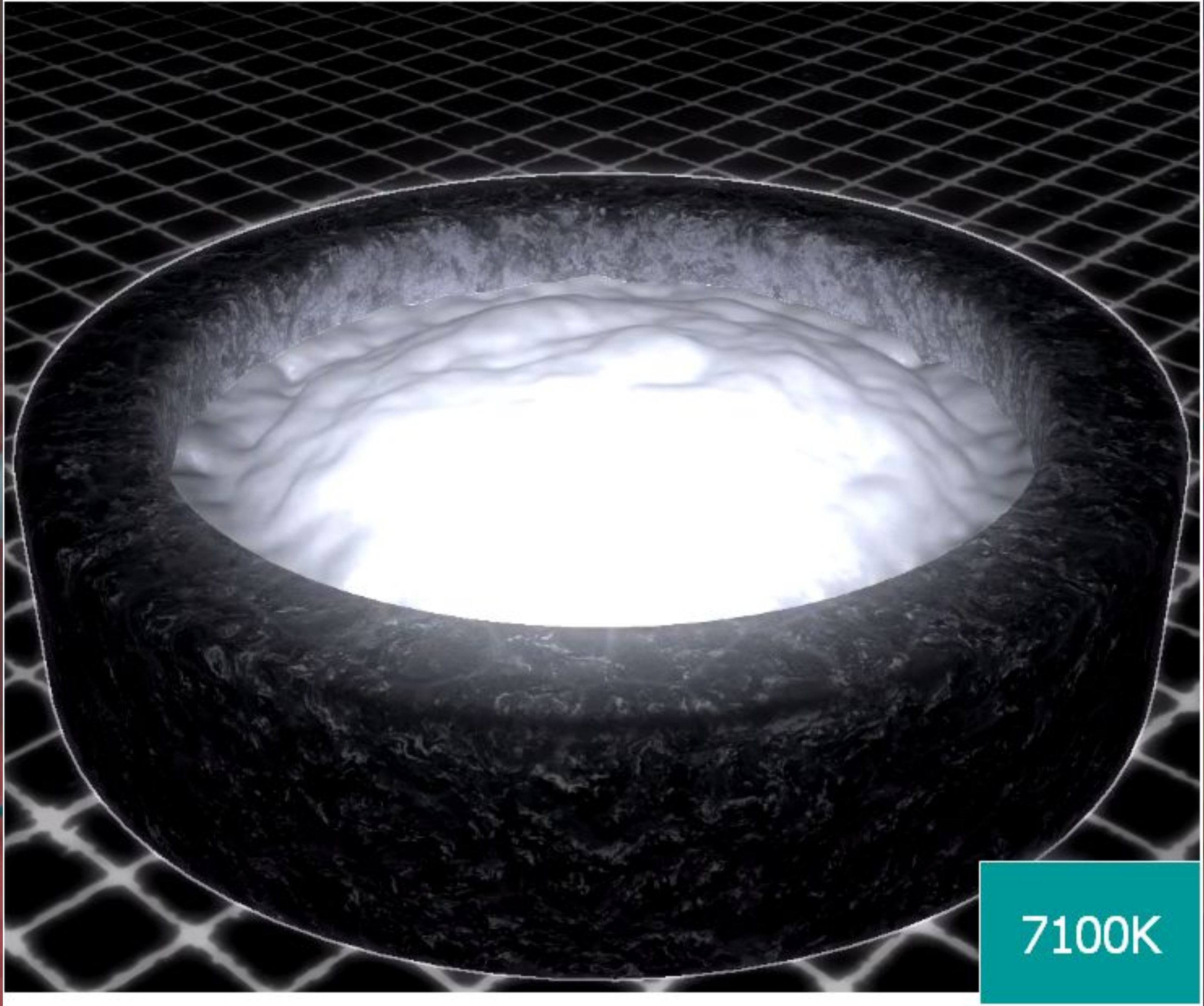
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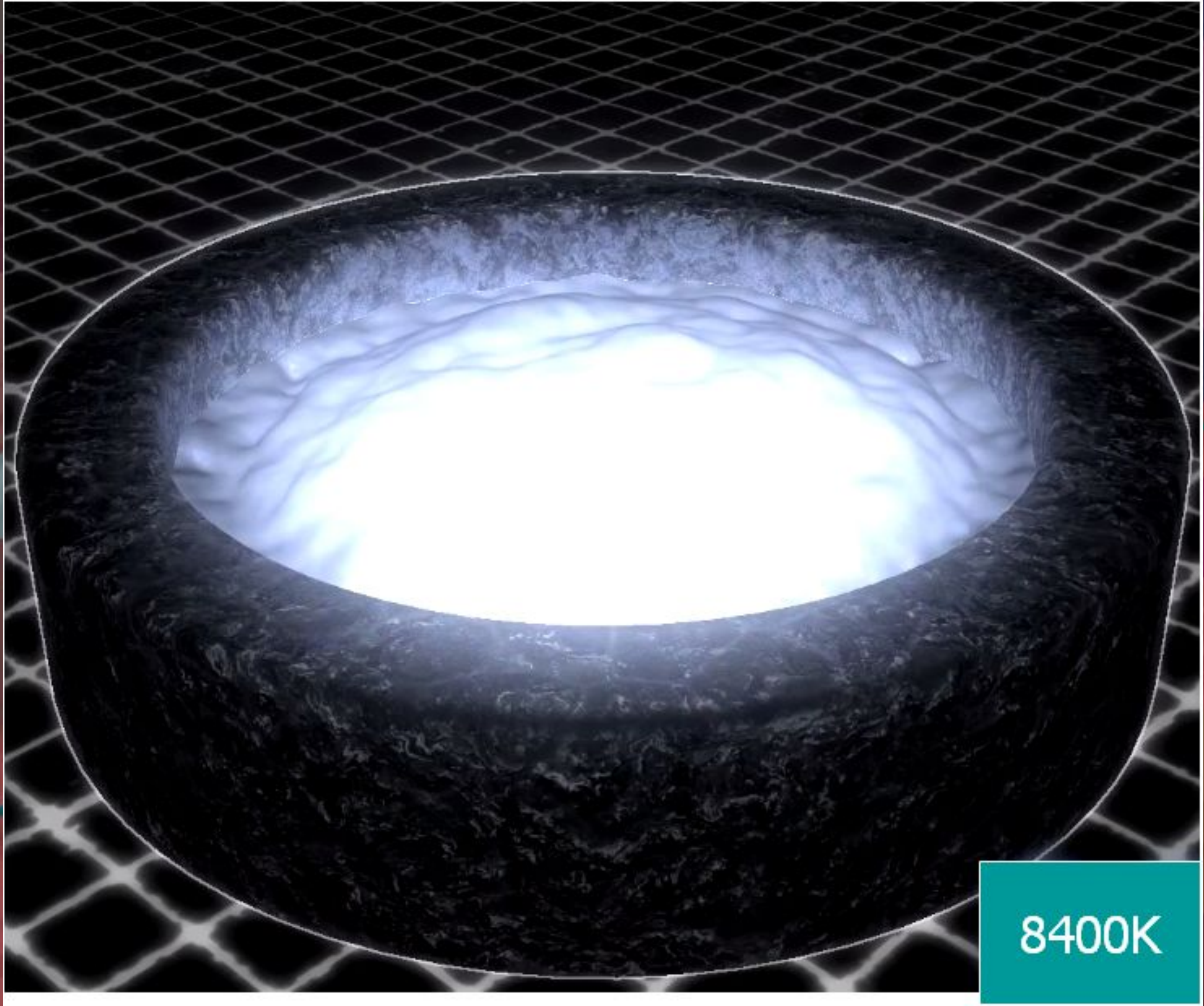
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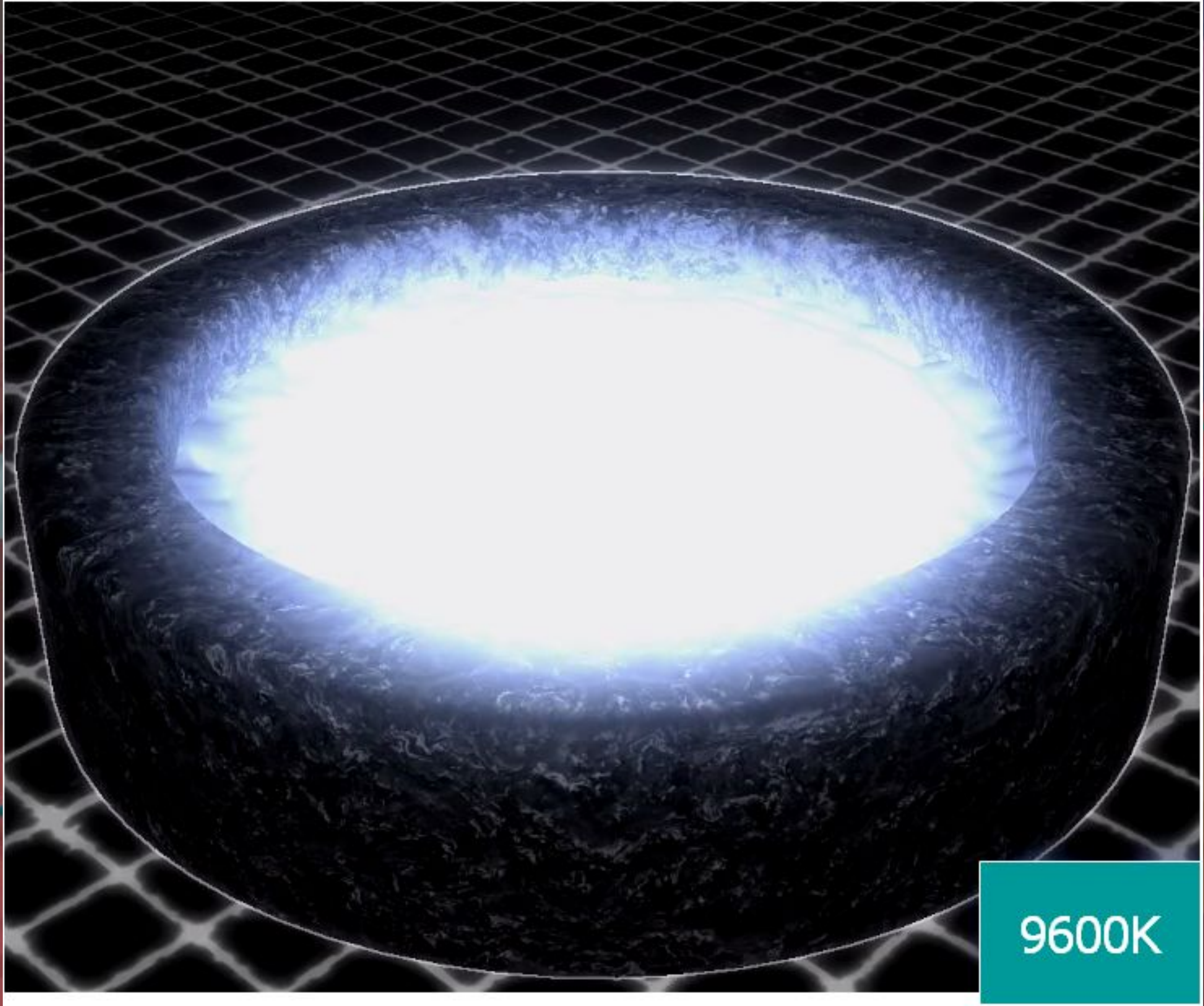
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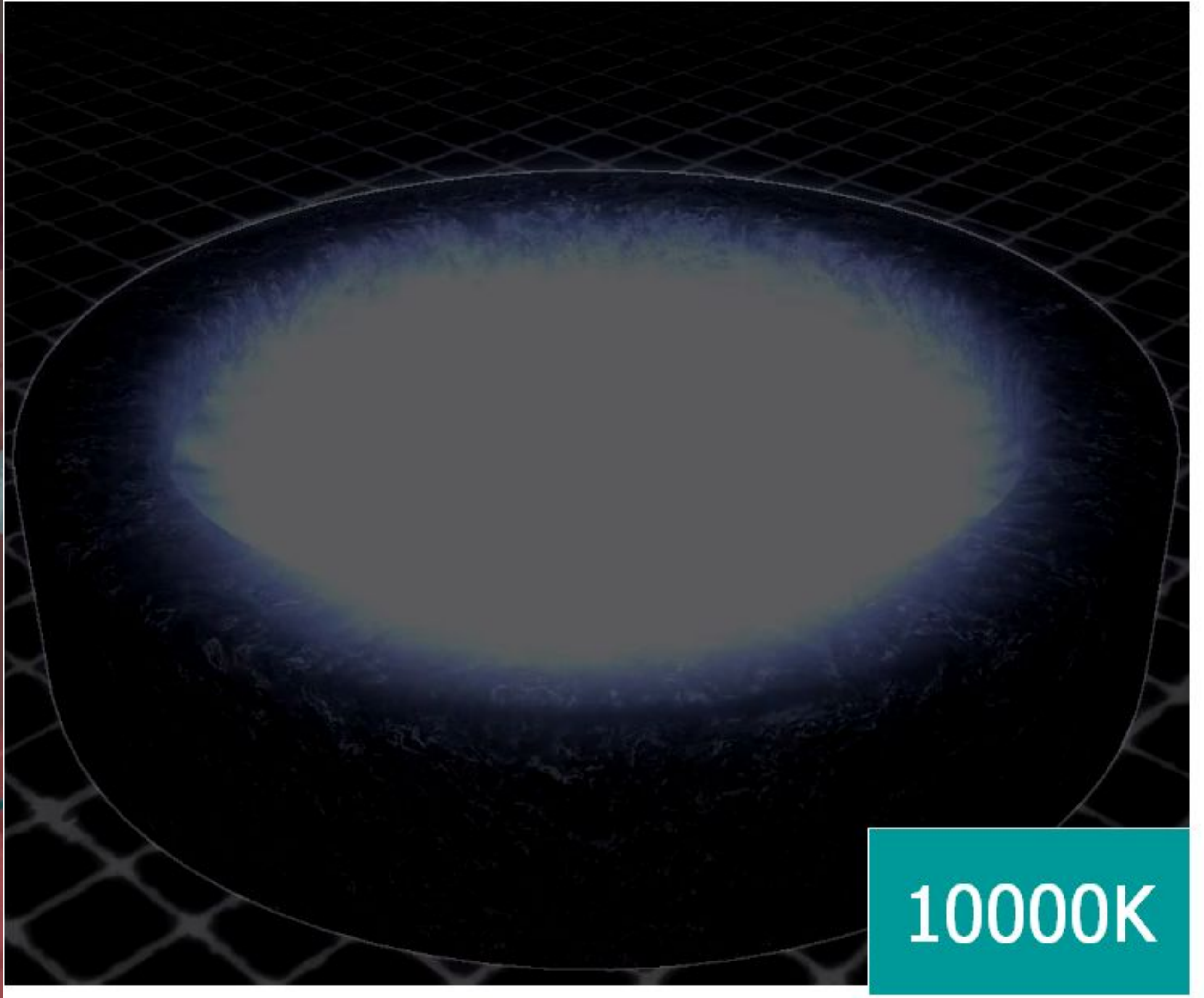
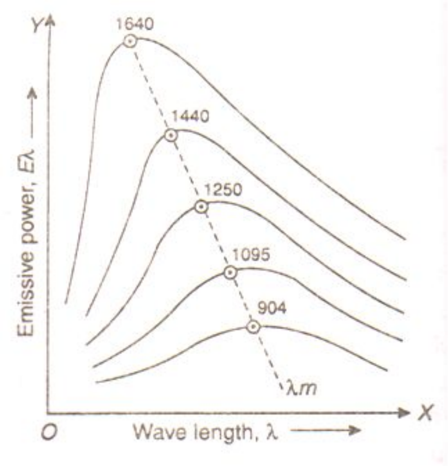

10000K

Plot of Experimental data:



That means T rises the radiation contains more and more short wavelengths. I.e. as the temperature is raised, the maximum intensity of emission shifts towards the shorter wavelength side (in other words if T is increased, λ_m decreases, that means the maximum intensity of emitted radiation is displaced towards the shorter wavelength side. This is actually **Wien's displacement law** giving,

$\lambda_m T =$ Constant (This is called Wein's displacement constant having value~ 0.2896 cmK)

Discussions with the pre 1900 classical theories

(i) **Wien's displacement law**: As shown in figure the wavelength corresponding to maximum energy represented by the peak of the curve shifts to shorter wavelength side as the temperature increases showing thereby that $\lambda_m T$=constant.

This confirms Wien's displacement law.

(ii) **Stefan's law**: The total energy of radiation at any temperature is given by the area between the curve corresponding to that temperature and the horizontal axis. The area is found to be proportional to 4th -power of the corresponding absolute temperature. This verifies Stefan's law.

(iii) **Rayleigh-Jeans law**: Wien's relation for distribution of energy with wavelength agrees with the experimental curves for short wavelengths but for long wavelengths there is a deviation. Lord Reyleigh and later on Jeans, therefore, tried to determine the form of the relation for distribution of energy with wavelength (on the assumption that the electromagnetic radiation emitted by a black body continuously vary in wavelength from zero to infinity). This radiation is considered to be broken up into monochromatic wave trains and the number of such wave trains within the range λ and $\lambda + d\lambda$ is determined by applying the law of probability. The expression comes out to be

$$dE = 8\pi\lambda^{-4} kT d\lambda$$

In the R & J treatment

$$dE = 8\pi\lambda^{-4}kTd\lambda => \rho(\lambda, T)d\lambda = 8\pi\lambda^{-4}kTd\lambda => \rho(\lambda, T) = 8\pi\lambda^{-4}kT$$

$\rho(\lambda, T)$ is the energy density

*(It can be proved easily. The proof is not given in my literature)

§ The Ultraviolet Catastrophe

Drawbacks of R & J law

(i) There is no maximum in the predicted spectrum (as a function of wavelength) $\Rightarrow$ this is not compatible with Wien's law.

(ii) Integration over λ will give the total energy (or total power) radiated by a black body but it clearly diverges $\Rightarrow$ not compatible with Stefan-Boltzmann law.

(iii) That means all black bodies are predicted to radiate with infinite energy (or power) due to the divergence if we go below and below UV ($\lambda \rightarrow 0$). So in a intensity-frequency plot, area under the curve tends to infinity. For any matter even a piece of chalk can emit UV, X-ray or γ-ray or something horrible like them but it can not be. Bad day came for the scientists. This discrepancy is called UV catastrophe.

§ Removal of UV catastrophe: Plank's Treatment

In1900 Plank suggested that the UV catastrophe could be fixed if energy comes in discrete chunks instead of being continuous.

Chunk size:

$$E = h\nu = hc/\lambda$$

In the cavity, in the wall, atoms are vibrating in different modes. Following Maxwell-Boltzmann statistics, the probability of having an energy *En* in a particular mode say, the *nth* mode can be written as,

$$P_n = Ae^{-E_n/kT} = Ae^{-nh\nu/kT}$$

After normalisation

$$P_n = Ae^{-nh\nu/kT} = (1 - e^{-h\nu/kT})e^{-nh\nu/kT}$$

So the average energy,

$$E_{av} = \sum_{n=0}^{\infty} E_n P_n$$

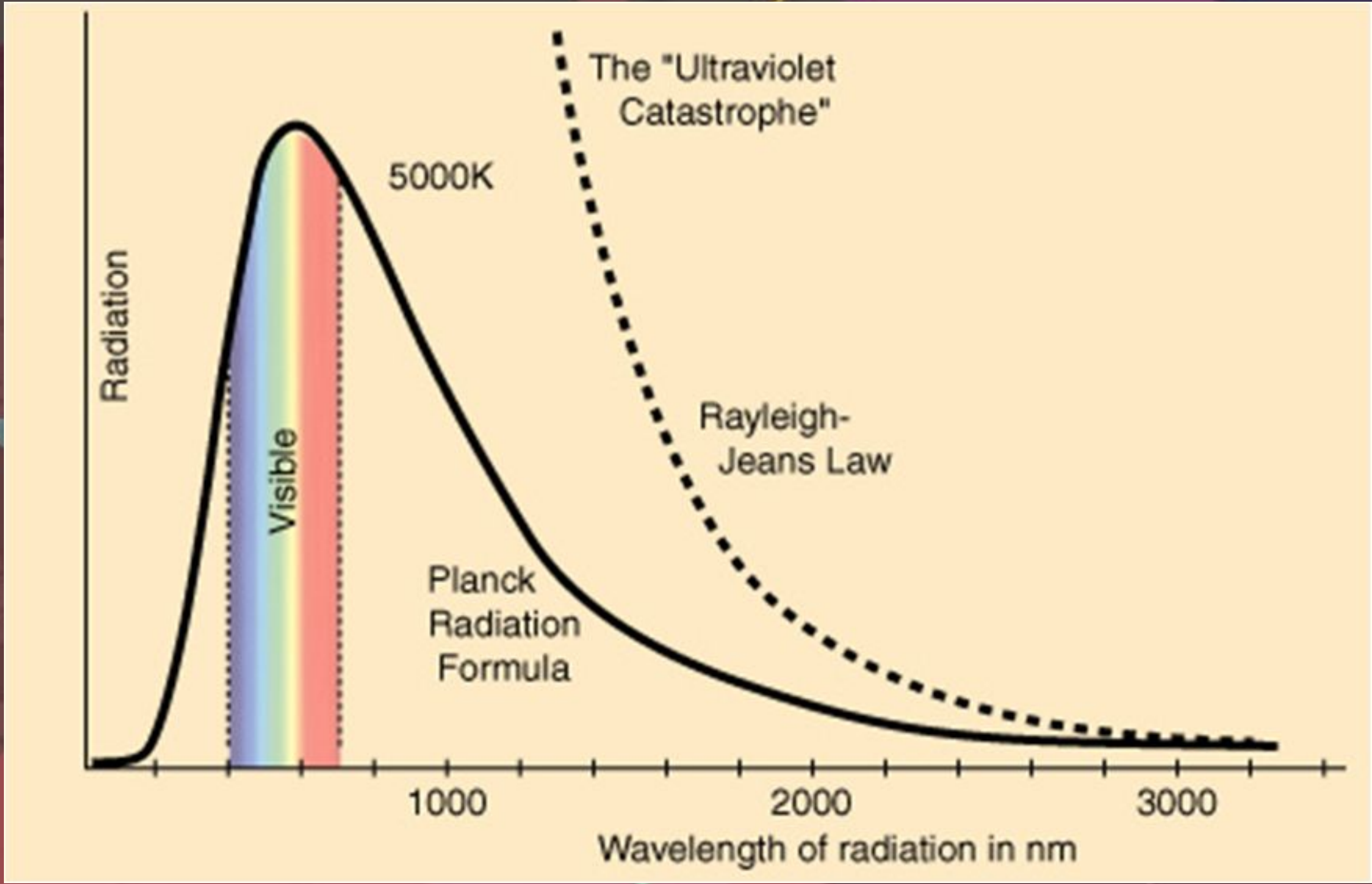
Finally it can be written as:

$$E_{av} = h\nu * e^{-h\nu/kT} / (1 - e^{-h\nu/kT}) = \frac{hc}{\lambda} \frac{(e^{\frac{-hc}{\lambda kT}})}{(1 - e^{\frac{-hc}{\lambda kT}})}$$

Instead of Putting average energy = kT If we put the average energy from Plank's treatment we get (energy of photon lies within the frequency interval per unit volume)

$$E(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{(e^{\frac{h\nu}{kT}} - 1)} => E(\lambda) = \frac{8\pi hc}{(e^{\frac{hc}{\lambda kT}} - 1)}\lambda^{-5} (numerically)$$

This is **Plank's law of black body radiation** in terms of wavelength


The "Ultraviolet Catastrophe"
5000K
Radiation
Visible
Rayleigh-Jeans Law
Planck Radiation Formula
1000
2000
3000
Wavelength of radiation in nm

§ Wien's law of blackbody radiation from Plank's law

Plank's law of blackbody radiation states that,

$|E(\lambda)| = \dfrac{8\pi hc}{e^{\frac{hc}{\lambda kT}} - 1}\lambda^{-5}$ Here symbols have their usual meanings.

Now for small λ, $e^{\frac{hc}{\lambda kT}} >> 1$. So the '1' at the denominator can be neglected.

$\Rightarrow |E(\lambda)| = \dfrac{8\pi hc}{e^{\frac{hc}{\lambda kT}}}\lambda^{-5} = A\lambda^{-5}e^{-\frac{hc}{\lambda kT}} = A\lambda^{-5}e^{-\frac{\alpha}{\lambda T}}$ [$A = 8\pi hc$ = a constant,

say and $\alpha = hc/k$, is another constant]

$\Rightarrow |E(\lambda)|d\lambda = A\lambda^{-5}e^{-\frac{\alpha}{\lambda T}}d\lambda$

This is Wien's law (applicable for short wavelength only).

§ Rayleigh-Jeans law of blackbody radiation from Plank's law

Plank's law of blackbody radiation states that,

$$|E(\lambda)| = \frac{8\pi hc}{e^{\frac{hc}{\lambda kT}} - 1}\lambda^{-5}$$ Here symbols have their usual meanings.

For long wavelength, $\frac{hc}{\lambda kT} < 1$

$$e^{\frac{hc}{\lambda kT}} \approx 1 + \frac{hc}{\lambda kT} \Rightarrow e^{\frac{hc}{\lambda kT}} - 1 \approx \frac{hc}{\lambda kT}$$

$$\therefore |E(\lambda)| = \frac{8\pi hc}{\frac{hc}{\lambda kT}}\lambda^{-5} = 8\pi kT\lambda^{-4}$$

$$\Rightarrow |E(\lambda)|d\lambda = 8\pi kT\lambda^{-4}d\lambda$$

This is Rayleigh-Jeans law (applicable for long wavelengths)

§ Wien's displacement law of blackbody radiation from Plank'slaw

Wien's displacement law states that the maximum intensity of radiation emitted by a black body is displaced towards shorter wavelength zone if the temperature is raised

Plank's law of blackbody radiation for λ states that,

$$|E(\lambda)| = \frac{8\pi hc}{e^{\frac{hc}{\lambda kT}} - 1}\lambda^{-5}$$ Here symbols have their usual meanings.

$$\Rightarrow \frac{d|E(\lambda)|}{d\lambda} = 8\pi hc \frac{d}{d\lambda}\left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\lambda^{-5}\right)$$

$$\Rightarrow \frac{d|E(\lambda)|}{d\lambda} = 8\pi hc\left(-5\lambda^{-6}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} + \lambda^{-5}\frac{1}{(e^{\frac{hc}{\lambda kT}} - 1)^2}e^{\frac{hc}{\lambda kT}}(\frac{hc}{kT\lambda^2})\right)$$

Now maximum intensity means maximum energy case.

$\therefore$ For maximum energy, $\Rightarrow \frac{d|E(\lambda)|}{d\lambda} = 0$

$$\Rightarrow R.H.S. = 0$$

$$\Rightarrow 8\pi hc\left(-5\lambda^{-6}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} + \lambda^{-5}\frac{1}{(e^{\frac{hc}{\lambda kT}} - 1)^2}e^{\frac{hc}{\lambda kT}}(\frac{hc}{kT\lambda^2})\right) = 0$$

T.O.

$$\Rightarrow e^{\frac{hc}{\lambda kT}}\left(1-\frac{hc}{5kT\lambda}\right)=1 \qquad \text{[do it yourself...]}$$

$$\Rightarrow e^{x}\left(1-\frac{x}{5}\right)=1 \qquad \text{[Putting } \left(\frac{hc}{kT\lambda}\right)=x\text{]}$$

$$\Rightarrow \frac{x}{5}=\left(1-\frac{1}{e^{x}}\right) \qquad \text{[Equation seems true for } x=0 \text{ but here } x\neq 0\text{]}$$

$$\Rightarrow x=\left(5-\frac{5}{e^{x}}\right)$$

$$\Rightarrow x=4.965$$

$$\Rightarrow \left.\frac{hc}{kT\lambda}\right|_{I=I_{max}}=4.965 \qquad \text{[Maximum intensity means writing } \lambda=\lambda_m\text{]}$$

$$\Rightarrow \lambda_m T=\frac{hc}{4.965k}=2.898\times 10^{-3}\, mK=\text{constant (called Wien's constant)}$$

This is Wien's displacement law.

§ Stefan's law of blackbody radiation from Plank's law

Stefan's law states that the radiation energy given out by a true blackbody per sec per unit area is directly proportional to the fourth power of the absolute temperature T.

$E_R \propto T^4$

$\Rightarrow E_R = \sigma T^4$ [σ is the constant of proportionality; called Stefan's constant having value $5.67\times10^{-8} Wm^{-2}K^{-4}$]

Consequence: If a black body at temperature T_2 is surrounded by another black body at temperature T_1, then the amount of energy radiated per unit time per unit area is, $E_{R'} = \sigma(T_2^4 - T_1^4)$

In terms of λ Plank's law of blackbody radiation (for energy magnitude) states that,

$$E(\lambda) = \frac{8\pi hc}{e^{\frac{hc}{\lambda kT}} - 1}\lambda^{-5}$$ Here symbols have their usual meanings.

In terms of frequency it ahs previously shown to have a form like below.

$$E(\nu) = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$\Rightarrow E(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}} - 1}$$

T.O.

If we want to calculate the total radiated energy for all frequency, then we have to integrate over all the frequencies. So integrating,

$$\int_0^\infty E(\nu)d\nu = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}}-1}$$

Putting $\frac{h\nu}{kT} = z\left(\Rightarrow d\nu = \frac{kT}{h}dz\right)$ the corresponding limits of the integral will be the same.

$$\Rightarrow \int_0^\infty E(\nu)d\nu = \frac{8\pi h}{c^3}\int_0^\infty \frac{\left(\frac{zkT}{h}\right)^3 \frac{kT}{h}dz}{e^z-1}$$

$$\Rightarrow E_R\big|_{Total} = \int_0^\infty E(\nu)d\nu = \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{z^3 dz}{e^z-1}$$

$$= \frac{8\pi k^4}{c^3h^3}T^4\int_0^\infty \frac{z^3 dz}{e^z(1-e^{-z})}$$

$$= \frac{8\pi k^4}{c^3h^3}T^4\int_0^\infty z^3e^{-z}(1-e^{-z})^{-1}dz$$

Here $z \geq 0 \Rightarrow e^{-z} \leq 1$ so, expanding $(1-e^{-z})^{-1}$ like $(1-x)^{-1} = (1+x+x^2+...)$ for $|x|<1$,

$$E_R\big|_{Total} = \frac{8\pi k^4}{c^3h^3}T^4\int_0^\infty z^3e^{-z}(1-e^{-z})^{-1}dz$$

$$= \frac{8\pi k^4}{c^3h^3}T^4\int_0^\infty z^3e^{-z}(1+e^{-z}+e^{-2z}+......)dz$$

T.O.

$$= \frac{8\pi k^4}{c^3h^3}T^4\int_0^\infty z^3(e^{-z}+e^{-2z}+e^{-3z}+......)dz$$

$$= \frac{8\pi k^4}{c^3h^3}T^4\left(\int_0^\infty z^3e^{-z}dz+\int_0^\infty z^3e^{-2z}dz+\int_0^\infty z^3e^{-3z}dz+......\right)$$

$$= \frac{8\pi k^4}{c^3h^3}T^4\left(\frac{6}{1^4}+\frac{6}{2^4}+\frac{6}{3^4}+.......\right) \quad [\because \int_0^\infty z^3e^{-kz}dz=\frac{6}{k^4}]$$

$$= \frac{8\pi k^4}{c^3h^3}T^4 6\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.......\right)$$

$$= \frac{48\pi k^4}{c^3h^3}T^4\sum_{i=1}^{\infty}\frac{1}{i^4} \quad [\sum_{i=1}^{\infty}\frac{1}{i^4}=\frac{\pi^4}{90}]$$

$$= \frac{48\pi k^4}{c^3h^3}T^4\frac{\pi^4}{90}$$

T.O.

$$= \frac{8\pi^5 k^4}{15c^3h^3} T^4 = AT^4$$

[A is a constant (total density radiation constant).]

So, the energy density of radiation inside a hollow enclosure is proportional to T^4. If we make a small hole on its wall, it can behave like a perfect blackbody. Again radiation per unit area per second from the hole is proportional to energy density within the enclosure. It is given by,

$$\frac{1}{4}c[E_R|_{Total}] = \frac{1}{4}c\frac{8\pi^5 k^4}{15c^3h^3}T^4 = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4$$

The σ is called the Stefan's constant and has a value $5.67 \times 10^{-5} erg / cm^2 \sec K^{-4} (= 5.67 \times 10^{-8} Wm^{-2} K^{-4})$

Some application areas

Isotherm Drawing on Earth
Radiation Telescopes
Heat Sensors & Imaging
Solar Energy
Lighting Systems

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Audience Q&A Session

ⓘ Start presenting to display the audience questions on this slide.

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Scientifically most valued law in Black-body Radiation is ………………… . Fill in the Blank.

ⓘ Start presenting to display the poll results on this slide.

Thanks To All

Add-ons used

1) Equation Editor++
2) Word Cloud Generator
3) Slido for google Slides